Evidence for Excellence in Education

Report

Evaluation of the IB Middle Years Programme Mathematics Skills Framework

National Foundation for Educational Research (NFER)

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Evaluation of the IB Middle Years Programme Mathematics Skills Framework

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Published in [October 2017]
By the National Foundation for Educational Research,
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ISBN 978-1-911039-52-5

How to cite this publication:
Hoon, N., Ellis, L. and Hopkins, A. (2016). *Evaluation of the IB Middle Years Programme Mathematics Skills Framework*. Slough: NFER.

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Executive summary

The International Baccalaureate (IB) commissioned the National Foundation for Educational Research (NFER) to evaluate the Middle Years Programme (MYP) Mathematics Skills Framework as part of the IB's curriculum review of MYP mathematics.

The IB's Middle Years Programme is designed for students aged 11 to 16. As of June 2016 the MYP is offered in over 1300 schools in over 100 countries. (The International Baccalaureate, 2016). The MYP provides frameworks of learning, rather than traditionally prescribed curriculums, to encourage concept-based teaching and learning (Erickson, 2007). The MYP Mathematics Skills Framework forms part of the *MYP Mathematics Guide* and sets out suggested content to support schools in structuring their own programmes of learning.

The current MYP Mathematics Skills Framework was launched in 2013 and is in its third year of teaching. The IB generally operates a seven year review cycle and as such the current framework is being reviewed for a new course to be launched between 2019 and 2020. Concurrent with the MYP Mathematics curriculum review, the IB also conducted a review of its Diploma Programme (DP) mathematics courses as part of its wider consideration of mathematics teaching and learning across IB programmes.

The research undertaken by NFER aimed to inform the current review cycle and the subsequent development of materials by IB staff and curriculum review participants.

The evaluation focused on addressing the following research questions:

Written Curriculum

1. How fit for purpose[1] is the MYP Mathematics Skills Framework?

   Do the four branches as specified reflect current thinking in educational research with regards to middle years education mathematical preparation for current and future learners?

   Is there sufficient breadth[2] across the four branches in the skills framework?

   Is there sufficient depth[3] in each of the four branches of the skills framework?

   Are there any gaps in skill coverage the written curriculum should address?

2. How well does the MYP Mathematics Skills Framework ensure a smooth transition to and links between IB Diploma Level courses, particularly Mathematics Standard Level and Higher Level?

Programme Implementation

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[1] Defined in Table 1.1 (page 3) as applied within this research project.
[2] Defined in Table 1.1 (page 3) as applied within this research project.
[3] Defined in Table 1.1 (page 3) as applied within this research project.

3. What are school perceptions of the MYP Mathematics Skills Framework?

   Do schools generally feel the MYP Mathematics Skills Framework has sufficient depth and breadth to meet the needs of current and future students?

4. How are schools and teachers using the MYP Mathematics Skills Framework in their planning?

   Is there sufficient information in the written curriculum to allow schools and teachers in differing contexts to plan horizontally and vertically for a robust mathematics programme in the middle years (ages 10-16)?

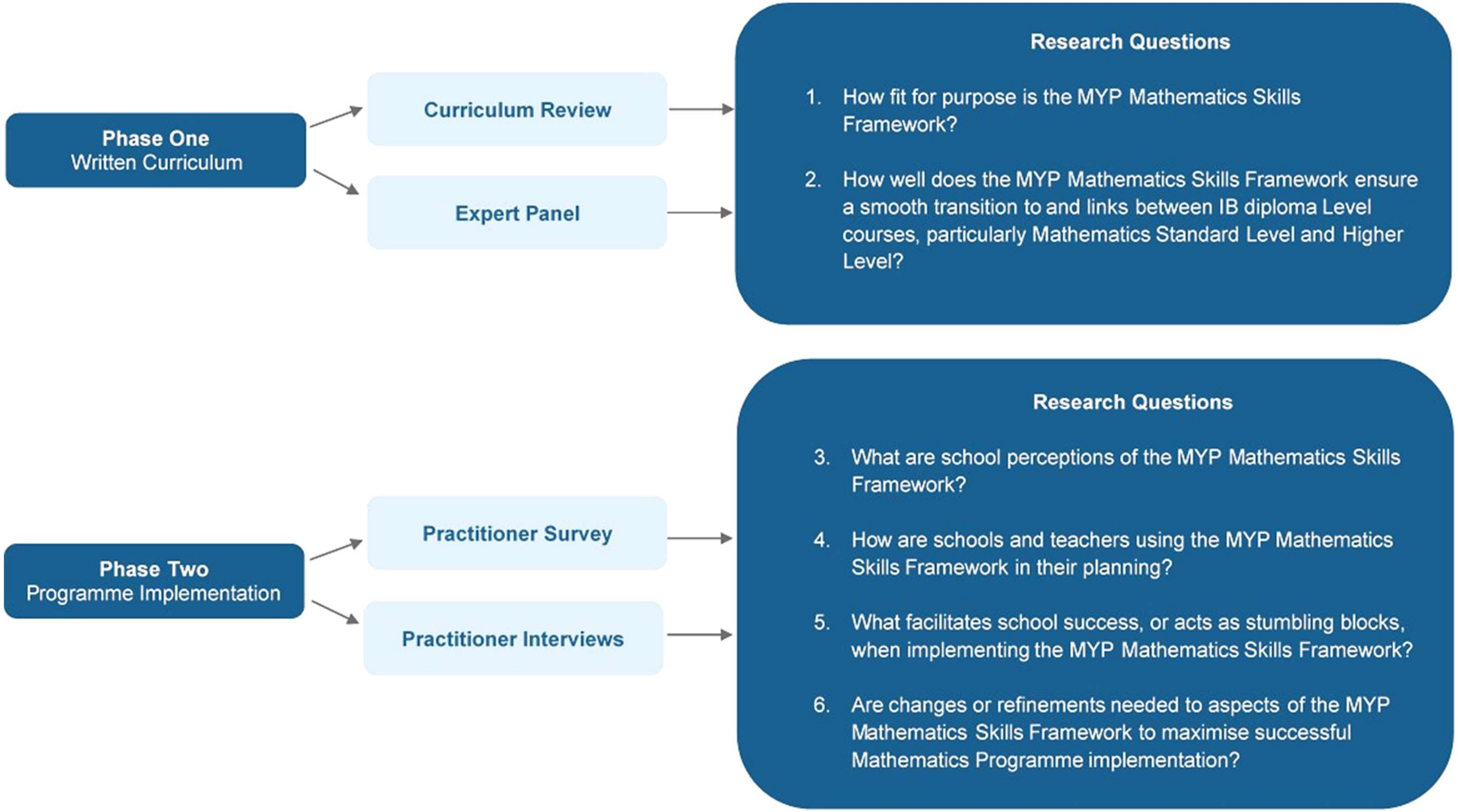
5. What facilitates school success, or acts as stumbling blocks, when implementing the MYP Mathematics Skills Framework?

   To what extent do schools perceive curriculum materials and related supports provided by the International Baccalaureate to aid MYP Mathematics Skills Framework implementation?

6. Are changes or refinements needed to aspects of the MYP Mathematics skills framework to maximize successful Mathematics programme implementation?

The NFER conducted a mixed methods study to answer these questions. This approach was chosen as it enabled multiple research activities to be carried out which could triangulate and increase confidence in the findings (Greene *et al*., 2005). The different research activities formed a two-phase approach and provided a rich source of both quantitative and qualitative data. Four main research activities were carried out: (1) curriculum comparison, (2) an expert panel, (3) a practitioner questionnaire and (4) practitioner interviews. An outline of the research activities is provided in Figure 1.

Figure 1: The two phases of the research activities



Summary of key findings

This section provides an overview of key findings from each research activity as they relate to the research questions.

Written curriculum

The curriculum comparison entailed mapping the MYP Mathematics Skills Framework against other national and international standards (see Section 2.1.1 for details). The curriculum comparison shows that there are many ways to structure a content framework for middle years mathematics learning. The current IB MYP Mathematics Skills Framework format of subdividing the content into four topics and providing guidance at two challenge levels follows a broadly similar approach to several of the other middle years systems in this study. A key difference between the IB approach and several of the other systems is that the IB does not provide specific or suggested year-by-year content. Instead it allows schools to structure this for themselves.

In terms of content specified, the IB MYP Mathematics Skills Framework contains broadly similar content to other middle years systems at the *Standard and extended* challenge level. At the *Extended* challenge level the IB MYP Mathematics Skills Framework contains a number of topics that are beyond the scope of other key middle years systems. Some of these *Extended* topics are not essential pre-requisites to access the IB Diploma Programme (DP) mathematics courses. Including these topics may lead to tension between breadth and depth of learning.

The expert panel echoed many of the findings of the curriculum comparison in terms of content. A general consensus from the panel was that depth of understanding often lies in a learner's ability to use and apply knowledge and skills to solve problems, reason intuitively and make connections between mathematical ideas. Within the IB *MYP Mathematics Guide*, these aspects are covered outside of the Mathematics Skills Framework, and hence the scope of this study, but we would recommend they should be considered as part of the IB review of middle years mathematics learning.

Programme implementation

The practitioner questionnaire[4] indicated predominately positive feedback about the MYP Mathematics Skills Framework, although there were concerns from some practitioners about the appropriateness of some topics and the level of planning support. Many practitioners indicated that the Mathematics Skills Framework allowed them to structure their mathematics curriculum for each year group, although embedding the content within the wider IB philosophies was seen as more of a challenge. Approximately 80 per cent[5] of practitioners agreed that the framework provided sufficient subject-specific support. The *Standard and extended* challenge level guidance was generally rated more highly as being appropriate and supportive.

[4] A total of 679 practitioners in 279 schools completed the survey. 518 responded to the full questionnaire and 161 provided partial responses.

[5] See Section 3 for detailed analysis of responses to the practitioner questionnaire.

A larger number of practitioners felt that more detailed written guidance was needed in some topics at the *Extended* challenge level.

The practitioner interviews[6] provided useful case-studies to follow up on the emerging themes from the practitioner questionnaires. In particular, they supported findings that the flexible and non-prescriptive nature of the MYP Mathematics Skills Framework can be used alongside other systems that schools may be required to deliver. However, some interviewees noted that this resulted in the guidance sometimes being too brief, and at odds with the more prescriptive and detailed format of the DP mathematics guides.

[6] Four practitioner interviews were carried out; two with practitioners in IBA, one in IBAEM and one in IBAP.

1 Introduction

1.1 Chapter outlines

This report is structured in sections relating to the two phases of the research activities; the written curriculum and programme implementation. The key findings from the two phases and recommendations are presented in the final chapter.

1.1.1 Chapter 2 Written curriculum outline

Each section of Chapter 2 contains findings from both the curriculum comparison and expert panel discussions. Section 2.1 describes the methodological approach taken in Phase 1. Sections 2.2 – 2.4 summarise the key findings as they relate to the research questions concerning breadth, depth and fitness for purpose of the current MYP Mathematics Skills Framework. Any gaps in coverage are also presented in these sections. Sections 2.5 and 2.6 summarise some specific issues that were identified and examples of innovative practice in these areas. Section 2.7 considers IB MYP Mathematics within the IB wider age continuum.

1.1.2 Chapter 3 Programme implementation outline

This chapter presents the findings from the second phase of the research study, the online questionnaire and the follow up interviews with IB MYP practitioners. Practitioners were asked for their opinions and experiences using the MYP Mathematics Skills Framework. Practitioners were also asked about their experiences of the IB continuum and the links between the PYP and DP programmes.

Sections 3.1 and 3.2 describe the research design and data analysis methodology for Phase 2. Section 3.3 presents the data in relation to each of the four key research questions within Phase 2.

1.1.3 Chapter 4 Triangulation of findings outline

This chapter begins by triangulating the findings from the two phases of the research in terms of the overall structure of the MYP Mathematics Skills Framework and the quality of the written guidance it contains. The chapter then presents suggestions for revisions to the topics and skills in the current MYP Mathematics Skills Framework and other topics and skills that the research suggests should be considered for inclusion.

1.2 Definitions

Table 1.1 shows definitions used within this research.

Table 1.1: Definitions

Terminology	Definition
Breadth of learning	The guidance is comprehensive enough in its coverage such that learners will be able to attain sufficient understanding at the end of a five year study to progress to further post-middle years mathematics learning. NFER paid particular attention to DP mathematics courses as a frame to understand breadth but also examined non-IB courses[7].
Depth of learning	The guidance is comprehensive enough such that a 16 year old could be reasonably summatively assessed on any of the topics in the skills framework by the end of a five-year study. This includes: • developing a suitable level of conceptual understanding • being able to apply topics in different situations • making connections between topics.
Fit for purpose	The guidance should allow MYP teachers to structure their own school mathematics programmes. This includes: • being able to identify subject content appropriate for a particular year group / ability level • being able to plan vertically to ensure continuity and progression from year 1 to year 5 and beyond • being able to plan horizontally, both within mathematics and within subject groups to plan the scope of learning for a particular year.
Challenge level	The subject content in the MYP Mathematics Skills Framework is divided into two challenge levels: • *Standard and extended* (S&E) – aims to give all students a sound knowledge of basic mathematical principles; supports DP standard level mathematics courses • *Extended* (E) – additional topics and skills to provide a foundation for students who wish to pursue mathematics courses at DP higher level.
Systems	The curriculum comparison research activity compared the MYP Mathematics Skills Framework to a variety of international curriculums and systems. These are referred to as 'systems' within this report.

[7] To illustrate breadth, Alcantara (2015) states that in DP Mathematics HL 'breadth' covers all topics in Algebra and Functions, includes study of parametric equations and curves in Geometry; requires other areas of Pure Mathematics (1 as a prerequisite), and makes one more optional; covers all Statistics topics; also provides Discrete Mathematics as an option.

2 Phase 1 Written curriculum

Chapter outline

Each section of Chapter 2 contains findings from both the curriculum comparison and expert panel discussions. Section 2.1 describes the methodological approach taken in Phase 1. Sections 2.2 – 2.4 summarise the key findings as they relate to the research questions concerning fitness of purpose for teacher planning and the breadth and depth of the current MYP Mathematics Skills Framework. Any gaps in coverage are also presented in these sections. Sections 2.5 to 2.6 summarise some specific issues that were identified and examples of innovative practice in these areas. Section 2.7 considers IB MYP Mathematics within the IB wider age continuum.

2.1 Research methodology

For Phase 1 both a curriculum comparison and expert panel discussion were carried out. International representation of the curriculums, and on the expert panel enabled the researchers to gain a global perspective of middle years mathematics teaching and current academic thinking with regards to the future of middle years mathematics.

The research activities in Phase 1 were designed to address research questions (1) and (2):

1. How fit for purpose is the MYP Mathematics Skills Framework?

   Do the four branches as specified reflect current thinking in educational research with regards to middle years education mathematical preparation for current and future learners?

   Is there sufficient breadth across the four branches in the skills framework?

   Is there sufficient depth in each of the four branches of the skills framework?

   Are there any gaps in skill coverage the written curriculum should address?

2. How well does the MYP Mathematics Skills Framework ensure a smooth transition to and links between IB Diploma Level courses, particularly Mathematics Standard Level and Higher Level?

2.1.1 Curriculum document analysis

The IB had previously carried out internal research by programme development staff which provided a foundation for the curriculum comparison work to be carried out by NFER. As part of this prior research, a curriculum map had been created, mapping the current IB topics against:

- The US Common Core State Standards
- The Pearson GCSE curriculum in England
- The Cambridge iGCSE
- Singapore Mathematics courses for Secondary 1 to 4
- The Australia ACARA framework
- The Quebec Education Program

This curriculum map linked each statement from these systems to the topics included in the current MYP Mathematics Skills Framework. Any additional topics from the other systems that did not link to the topics in the MYP Mathematics Skills Framework were also listed.

NFER and the IB agreed that these systems were appropriate for the NFER curriculum comparison study. Collectively they include a wide range of systems representative of IB World Schools. The systems also provided a range of approaches to structuring guidance for mathematics learning. Additional curriculum documents linked to the key systems were also consulted where the systems do not span the full middle years age range, (e.g. in England, the national curriculum documents for key stages 3 and 4). This helped gain a better perspective on key middle years mathematics learning priorities.

To gain an overall understanding of the systems, NFER analysed the overall structure and format of each system. The structure of a system was considered in terms of age range, how the written practitioner guidance for course planning was subdivided, the approach to providing written guidance and the use of challenge levels or tiers.

The next stage of NFER's analysis involved using the IB curriculum map. NFER classified the MYP Mathematics Skills Framework topics into three groups:

- Group A: the topic is a feature of several other middle years systems, with similar levels of written support suggested by the number of curriculum references
- Group B: the topic is a feature of several other middle years systems, but other systems have significantly more curriculum statements relating to this topic than suggested by the MYP Mathematics Skills Framework topics and skills statements
- Group C: the topic is not a key feature of the majority of other middle years systems.

Classifying the IB topics into these groups allowed NFER researchers to then examine the topics in groups B and C in greater depth. This content was grouped into themes to more readily allow for comparison against IB documentation.

Some of the topics in group C were features of only the *Extended* challenge level MYP Mathematics Skills Framework. These topics were compared to a wider range of systems to help consider their suitability within the middle years:

- England AQA level 2 Further Mathematics
- US Common Core State Standards for High School
- ACARA Year 10 Extension
- Singapore – Additional Mathematics courses

These systems were selected as they closely link to the some of the key systems used in main IB curriculum comparison study and provide guidance for directing middle years learning for mathematically more-able students.

In the curriculum mapping, some content from the other systems could not be linked directly to the topics listed in the MYP Mathematics Skills Framework. NFER grouped this content into themes and analysed these themes against IB Primary Years Programme (PYP) Mathematics Scope and Sequence documentation.

To analyse the continuity between the middle years and Diploma Programme (DP), (research question 2) the prior learning expectations in the DP documentation were consulted. This was compared to the MYP Mathematics Skills Framework to consider the depth and breadth of topic coverage in the MYP Mathematics Skills Framework against pre-requisite needs for DP courses. Key topic areas that were missing from the MYP Mathematics Skills Framework but were stated as prior learning expectations were identified and drawn out of the documentation for discussion.

In addition to analysing the MYP Mathematics Skills Framework, NFER also considered the full *MYP Mathematics Guide* and the *MYP: From principles into practice* document. NFER examined these documents to help contextualise the MYP Mathematics Skills Framework and to better support conclusions about the fitness for purpose of the MYP Mathematics Skills Framework, particularly in terms of breadth and depth of learning. This also provided NFER researchers with a clearer understanding of the IB approaches and philosophies to teaching and learning to help contextualise discussions during the expert panel and to inform Phase 2 research activities.

2.1.2 Expert panel

To support the curriculum comparison desk research, NFER and the IB agreed that additional subject expert specialist input would provide a greater depth of understanding of priorities in middle years mathematics education on an international scale. An expert panel discussion was organised to allow NFER to bring together a range of panel members, each with particular areas of specialist knowledge and expertise. The 'participant-to-participant' nature of an expert panel discussion, (Hartas, 2010), was considered advantageous to the particular nature of this research project, in which it was unlikely that any given expert would be positioned to have an in-depth understanding on both an academic basis and an understanding of the IB philosophies. As such, NFER and the IB agreed the panel should comprise of three academic experts with a background in middle years curriculum design and two

IB MYP mathematics practitioners who could help contextualize IB MYP Mathematics.

NFER consulted subject and professional organisations and surveyed academic literature on middle years mathematics curriculum development to identify a range of potential academic experts, which was then shared with the IB. The IB also identified a range of potential academic experts and provided NFER with a list of suitable IB MYP practitioners, who as well as being familiar with MYP mathematics, also held other roles within the IB mathematics system. A stepwise approach was taken to finalise a shortlist of candidates to the IB and from this shortlist NFER and the IB agreed the final selection. Collectively, these five experts covered the three IB regions (Americas, Europe, Africa and the Middle East, and Asia Pacific). Local perspectives and cultural differences were minimised by selecting participants with a broad range of experience and across a range of geographical locations. Further detail about the expert panel, including brief profiles of each expert, is provided in Supplementary Report 1 – Expert panel high level report. See Appendix A.

The main panel discussion was held on July 15th 2016 with four experts, and a subsequent individual discussion was held on August 15th with an academic expert from Australia. Prior to the main panel discussion, participants were sent a pre-panel discussion packet which included;

- The current *MYP Mathematics Guide*
- Extracts from the PYP and DP guides indicating key subject content
- An extract from the 2008 *MYP Mathematics Guide* indicating the content in the Discrete Mathematics branch as this was a topic explicitly of interest to IB curriculum staff.
- Summary information about the IB, NFER and the research project
- Key information / questions relating to issues arising from the curriculum comparison work.

As well as providing the experts with contextual information about IB MYP mathematics and the research project, key questions were provided within the pre-panel materials to allow the experts time before the discussion to consider their thoughts. The focus for these questions included:

- What are the needs of current and future mathematical middle years learners?
- What are the current priorities and innovations in middle years mathematics education?
- Expert opinion on the breadth, depth and fitness-for-purpose of the current IB MYP Mathematics Skills Framework.

Specific content questions were also asked, linked to NFER's initial findings from the curriculum comparison work. These questions focussed on:

- Ratio, percentages and proportion
- Functions, graphs and transformations

- Statistical analysis
- Digital technologies
- Discrete mathematics
- Development of mathematical skills / fluency.

The experts were also sent a pre-panel questionnaire. For each branch the experts were asked to state whether they thought the current topics listed in the MYP Mathematics Skills Framework were relevant to the middle years, and if so at what challenge level. In addition, the panel was asked for any other topics they thought may be relevant but were currently not included in the framework. These questionnaires were returned to NFER prior to the panel to help identify key discussion points for the panel. A summary of the responses are provided in Appendix A Expert panel report

The discussion was recorded after informed consent was obtained by the participants to facilitate accurate summaries of key points. The key questions within the pre-panel reading material provided a framework for writing a high-level report of the discussion, and are provided in Appendix A – Expert Panel Report. In addition, further themes emerged from the discussion concerning how the framework sits within the full *MYP Mathematics Guide*, and how it may be used in practice. The expert panel notes were reviewed to identify key themes, and then the recording was used to summarise the discussion threads. The experts reviewed the report to validate that it provided a representative summary of the discussions.

A second discussion was carried out as one of the academic experts was unable to attend the main panel. NFER used a similar approach for this second expert discussion. NFER asked similar general questions about priorities for mathematics learning / education, breadth and depth and adapted the pre-panel questions to allow a more granular discussion of the content and skills within the framework itself. NFER used the same format as the first expert panel report to summarise this second discussion. As described in the previous paragraph this expert was given the opportunity to review the written report for validity of representative findings.

2.1.3 Case studies and supporting research literature

To support the main research activities of Phase 1, NFER conducted a review of key research literature and innovative practice that were raised during the expert panel discussions. High-level summaries are presented alongside the evidence for each of the research questions as case studies. Each case study provides a brief overview; references and links to further reading are supplied in the 'Further Reading' section at the end of this report.

2.2 Fit for purpose? The overall structure of the IB Middle Years Programme Mathematics Skills Framework

2.2.1 Structures of systems

A fundamental aspect of the fitness-for-purpose of the MYP Mathematics Skills Framework is whether the approach of dividing content across four branches and two challenge levels reflects the priorities of current thinking in mathematics education. As part of the phase 1 research activities, NFER compared the structures of each of the systems. NFER was able to triangulate this structural analysis with other key research activities, including the more granular analysis of content coverage, the expert panel discussions and the practitioner questionnaires, to develop an in-depth analysis of the overall fitness-for-purpose of the current MYP Mathematics Skills Framework.

A key feature of all of the systems is that they structure content into broad branches or strands. Table 2.1 summarizes these structures. Although each system uses its own specific structures, there is a general trend to divide the curriculum into strands involving number (including number systems, arithmetic skills and proportionality), algebra, geometry/measures, and statistics/probability. Some systems group number and algebra together reflecting the development of algebra within middle years learning as a construct to generalise about number.

A key difference of the systems is the approach to dividing content by challenge level or age, as detailed in Table 2.1. Several systems, including the MYP Mathematics Skills Framework, divide content across two or more challenge levels with an expectation that practitioners should decide on the appropriate challenge level for each learner to work from. Several systems instead adopt a year-by-year structure, prescribing learning content for each year group within the middle years.

Table 2.1: Structures of key systems in comparison study

Middle years system	Structure		
	Branch / Strand structure	**Branch / strand names**	**Challenge levels / tier / suggested age groups**
IB MYP	4 branches, subdivided into topics and skills	Number Algebra Geometry and trigonometry Statistics and probability	Ages 11 to 16 Two tiers Standard and extended – all students Extended – more-able students
England	5 topic areas	Number Algebra	Ages 14 to 16 Two tiers Foundation – all students

Middle years system	Structure		
	Branch / Strand structure	**Branch / strand names**	**Challenge levels / tier / suggested age groups**
Pearson Edexcel GCSE (9-1)		Ratio, proportion and rates of change Geometry and measures Statistics and probability	Higher – more-able students
Cambridge iGCSE	9 topics	Number Algebra and graphs Geometry Mensuration Co-ordinate geometry Trigonometry Matrices and transformations Probability Statistics	Ages 14 to 16 Two tiers Core curriculum – all students Extended curriculum – more-able students
Singapore Mathematics (Secondary One to Secondary Four)	3 strands, each subdivided into sub-strands, indicating content and learning experiences	Number and algebra Geometry and measurement Statistics and probability	Ages 12 – 16 5 different curriculum challenge levels (*): Mathematics O-Level, N(A)-level and N(T)-level Additional Mathematics N(A)-level and N(T) level Content organized by year: Secondary One Secondary Two Secondary Three/Four
US Common Core State Standards **(Grade 6 - 8)**	Each grade focuses on 5 domains Each domain is made up of standards	Ratios and proportional relationships (Grade 6 and 7 only) The number system Expressions and equations Functions (Grade 8 only) Geometry Statistics and probability	Ages 11 to 14 Content organized by grade
Australian Curriculum **(ACARA)** **(Year 7 – 10)**	3 strands, each sub-divided into threads	Number and algebra Measurement and geometry Statistics and probability	Ages 12 to 16 Content organized by grade

Middle years system	Structure		
	Branch / Strand structure	**Branch / strand names**	**Challenge levels / tier / suggested age groups**
Quebec Education Program	3 broad topic areas	Arithmetic and algebra Statistics and probability Geometry	Age 12 to 16 Secondary cycle one Secondary cycle two (three possible pathways)

*Within the Singapore curriculum, there is opportunity to move between each of the curriculums throughout the years, with some overlapping content across each curriculum

For each system, the structure seems in part influenced by the age range that the curriculum documents span, and has its own features which impact the approach to learning mathematics. For example, as the full ACARA curriculum spans a wide age range, from foundation (age 5) to Year 10 (age 16) having a single stand for *Number and algebra* helps develop the use of algebra as a generalised form of number, and allows for written guidance to connect ideas such as the associative, commutative and distributive laws as they apply to both branches.

> *Number and algebra are developed together, as each enriches the study of the other. Students apply number sense and strategies for counting and representing numbers. They explore the magnitude and properties of numbers. They apply a range of strategies for computation and understand the connections between operations. They recognise patterns and understand the concepts of variable and function. They build on their understanding of the number system to describe relationships and formulate generalisations. They recognise equivalence and solve equations and inequalities. They apply their number and algebra skills to conduct investigations, solve problems and communicate their reasoning.*
>
> **(ACARA, 2015)**

During the expert panels, participants were asked for their views on how fit for purpose the current MYP Mathematics Skills Framework is. Two themes emerged from the panel discussion. First the discussion centred on the need to make links and connections between different mathematical ideas / topics clearer than how the framework itself is structured. One expert commented that subdividing it further may increase the risk of a ‘compartmentalized curriculum’ which may not support the richness of making connections across branches. Secondly, experts discussed the level of guidance and written support for teachers to be able to structure their own mathematics programmes. A key aspect of this was how to provide useful support and guidance to practitioners without becoming over-prescriptive. The experts were collectively more familiar with the systems used in the US, Australia and New Zealand, and commented on how the different formats of the guidance provided by each system and the use of online materials to support practitioners. A brief summary of how these systems provides guidance on subject content is presented below as a set of short case studies.

Case Study: US Common Core State Standards for Mathematics

The online documentation can be accessed in several formats. The standards can be easily downloaded as a document presented by grade. Within the guidance for each grade, some general critical learning areas are presented. These are followed by a single-page overview of the key content for the grade presented by each of the content domain areas. Each of these overviews is then broken down into greater detail.

Within the online documentation, the standards can also be viewed by domain. This allows practitioners to see how different content and skills progress throughout the grades.

Case study: Australian Curriculum (ACARA)

The online documentation can be accessed in multiple formats. A matrix-design scope and sequence document showing content against year group allows practitioners to see how skills within a topic progress throughout the years. The guidance is also provided in a year-by-year format, also providing a high-level description of what the key proficiencies (understanding, fluency, problem-solving and reasoning) look like for the particular year group.

An online curriculum filter is provided allowing practitioners to access further detail about each curriculum statement, including elaborations of what the content may look like in practice, cross-curricular priorities and annotated work samples to help practitioners develop a deeper understanding of the ACARA proficiencies.

Case Study: The New Zealand Curriculum Online

The online documentation is provided in multiple formats. Practitioners can access achievement objectives for mathematics and statistics by learning area or by level. These levels are linked to the National Standards, setting out what pupils should be able to do at different ages.

The ‘National Standards’ area provides multiple guidance documents. Within this, ‘The standards’ area sets out what pupils are expected to able to do, together with examples of questions that exemplify the thinking and response strategies to exemplify whether the standard has been achieved.

2.2.2 Quality of written guidance

During the curriculum document review work, NFER noted a significant difference in the underlying design, structure and language of the different systems. The MYP Mathematics Skills Framework is presented as a set of topics that are suggested content, supported by skills that provide some further elaboration. The MYP Mathematics Skills Framework also relates each branch to the wider IB concepts and to post-MYP study. A fundamental design element of the MYP Mathematics Skills Framework is for teachers to be able to use the guidance flexibly to construct their own programmes of study. Furthermore, the content in the MYP Mathematics Skills Framework is provided as a list of examples as opposed to a prescribed curriculum.

This approach is significantly different to the majority of other systems. Some of the other systems (e.g. US Common Core State Standards, ACARA) are presented in year-by-year programmes. Most of the other systems also provide greater written detail about what specific skills a learner should be able to demonstrate, either by a particular year or phase of the middle years age range. In some cases (Pearson GCSE, Cambridge iGCSE), this may be directly linked to the system being intrinsically linked to an external end-of-course assessment, which may require greater detail in the written documentation.

During the expert panel there was a wider discussion concerning pre-requisite skills needed at a practitioner level in order to be able to use the IB MYP Mathematics Skills Framework effectively. Within the wider *MYP Mathematics Guide*, the *Planning the mathematics curriculum* examples provided (pages 16 – 17) were commented on positively. One expert praised the *MYP Mathematics Guide* as a whole, commenting that it was a very well-written set of documents. Another commented on the change in tone of the language used in the IB *MYP Mathematics Guide* as a whole and the MYP Mathematics Skills Framework itself, with the latter seeming far more procedural and traditional. Positive comments were made about the wider support from the IB, e.g. professional development events, that help practitioners to embed the MYP Mathematics Skills Framework content within the wider IB philosophies and approaches to learning. Concern was raised, however, that in reality practitioners may use the MYP Mathematics Skills Framework outside of the desired context, and simply view it as a list of isolated topics to work through with little thought to what content is appropriate to different ages or abilities or to the links between mathematical ideas or to the wider ideas of conceptual learning / global contexts. If used in this way, much of the richness of learning, which in turn forms a key aspect of depth of mathematical understanding, may be lost. If viewed as a 'tick list' of content, there is a danger that only superficial learning may be developed. There is a sense in which the middle years sees a change in emphasis in mathematical learning, with learners gaining a more sophisticated understanding of increasingly abstract ideas; this principle is embedded in the IB MYP philosophy of conceptual learning, but is not evident within the MYP Mathematics Skills Framework itself.

2.3 Breadth of content coverage

2.3.1 Curriculum mapping comparison

During the curriculum comparison work, the IB curriculum mapping was used to classify IB MYP mathematics topics into three groups:

- Group A: topic is a feature of several other middle years systems, with similar level of written support suggested by curriculum references
- Group B: topic is a feature of several other middle years systems, but other systems have significantly more curriculum statements relating to this topic than IB MYP

- Group C: IB MYP Mathematics Skills Framework contains skills that go beyond expectations of other systems or topic is not a key feature of the majority of other middle years systems

Due to the different curriculum structures, some topics / skills appeared in different places within different systems, e.g. sequences and exponents / laws of indices that span both number and algebra. Table 2.2 summarises the findings of this classification process.

Table 2.2: Number of IB MYP framework topics per branch in each of the three classification groups

IB Branch	Number	Algebra	Geometry and trigonometry	Statistics and probability
Group A	9	10	12	6
Group B	1	0	0	0
Group C	3	6	4	1
Total	**13**	**16**	**16**	**7**

This analysis indicates that the current MYP Mathematics Skills Framework provides a breadth of learning that is broadly in-line with other middle years systems. Overall during the expert panel activities, there was general agreement that the majority of topics within the MYP Mathematics Skills Framework were suitable for middle years learners. The only topic in which the breadth suggested by the IB MYP Mathematics Skills Framework seemed significantly different to other systems was *Ratio, percentage; direct and inverse proportion*, within the *Number* branch. This was also commented on by the expert panels, both in terms of breadth and depth, and we have reported on this in greater depth in section 2.5. The *Statistics and probability* branch and particularly the statistics elements were also commented on in terms of both breadth and depth, as discussed in section 2.6.

2.3.2 Topics in IB MYP but not in majority of other systems

The majority of the topics / skills within group C of the classification process were from the *Extended* challenge level of the MYP Mathematics Skills Framework. Table 2.3 provides a summary of whether these topics reflect content seen in the curriculums for the additional mathematics courses that might be taken by more able middle years learners, together with comments drawn out from the expert panel discussions. In some places topics / skills from different branches have been combined where there is a link between the content. A commentary is provided for each topic which summarises key comments from the expert panel and / or NFER's other research. More detailed commentary from the expert panel discussions can be found in Supplementary Report 1 – Expert panel high level report.

Table 2.3: IB MYP topics in groups C

Branch	IB MYP Topic	IB challenge level	Comment
Number	Sets	S&E	MYP specifies vocabulary / operations / properties of sets; in other systems some of these skills may be implicit. Some experts felt this topic may be better placed in the 'Statistics and probability' branch as, within the middle years, the skills are often used within the context of conditional probability.
Number / Algebra	Logarithms and number bases	E	Logarithms feature within several other courses for more able learners, with a similar level of demand to the MYP Mathematics Skills Framework. Experts commented that even at the *Extended* challenge level, with the main learning focussing on more procedural aspects of manipulation as opposed to conceptual understanding. Whilst the introduction of logarithms within the middle years may provide a sense of completeness (e.g. linking together exponents and inverse functions), there may be a question as to whether this topic promotes only superficial learning. Questions were raised as to the need for 'number bases' and whether this really supported learning. One expert commented on the potential impact of the optional IB eAssessment on how this topic is taught within the middle years.
Algebra	Algorithms	S&E	Other systems reference some specific procedures, e.g. use of compound interest formula / general iterative procedures. Experts felt that being able to use well-defined algorithmic processes efficiently is an important aspect of mathematical fluency.

Branch	IB MYP Topic	IB challenge level	Comment
Algebra	Inequalities / linear programming	S&E	Representing inequalities graphically does feature in other curriculums. Linear programming, in terms of setting up a set of inequalities to model a situation, and then using procedures to find an optimal solution does not seem to be an feature of other middle years systems, or of content in courses designed for more able students. The IB may benefit from clarifying the intended extent of the learning within this topic as it relates to the needs of middle years learners.
Algebra	Functions and transformations of functions	E	The IB MYP Mathematics Skills Framework included domain / range and suggests a higher level of demand in considering trigonometric functions, however this is commensurate with the demand seen in the courses for more able middle years learners. During the expert panel discussions, exponential functions was raised as a possible topic for inclusion. Within the middle years, this may be just as relevant as consideration of quadratic function, and perhaps even more so in terms of problems to do with growth and decay that are age-appropriate.
Algebra	Arithmetic and geometric series	E	Using / describing sequences is a feature of most middle years systems. Sums of infinite series is not in the main middle years curriculums, but does feature in some of the additional courses that may be taken by more able learners (e.g. AQA Level 2 Further Mathematics).
Geometry and trigonometry	Three-dimensional co-ordinate geometry	E	Although some other systems include consideration of three-dimensional problems (e.g. use of trigonometry / Pythagoras' theorem), three dimensional co-ordinate geometry does not feature explicitly even in curriculum documents for more able learners.

Branch	IB MYP Topic	IB challenge level	Comment
Geometry and trigonometry	Vectors and vector spaces	E	Experts questioned whether consideration of vector spaces was appropriate to the middle years. Whilst vector geometry offers an alternative method for solving geometrical problems, experts considered it was not necessary to include within the middle years, and learners would not be disadvantaged by not having considered it as part of a middle years course. Applying vectors to modelling the motion of particles features in the Singapore Additional Mathematics course. Vectors and matrices are a part of the US Common Core State Standards for High School.
Geometry and trigonometry	Trigonometric identities	E	This topic is contained within other courses for more-able learners.
Statistics and probability	Standard deviation	E	Expert opinion was that the idea of deviation, and then the concept of standard deviation as measure of dispersion, may even be suitable to consider at the *Standard and extended* challenge level. Calculation of standard deviation is best left at the *Extended* challenge level and does feature in other courses for more-able learners.

2.3.3 Topics in other systems but not in IB MYP

During the curriculum comparison, we identified some topics that are included in other middle years mathematics programmes, but are not explicitly stated in the MYP Mathematics Skills Framework, as presented in table 2.4. As with the previous table relevant expert panel and NFER comments are included to further explicate the theme for curriculum review purposes.

Table 2.4: Topics not explicitly stated in IB MYP

Theme	Comment
Use of timetables / 12 and 24 hour clock	This is both explicit and implicit from PYP, for example in measurement Phases 1 and 2 (p.90) as well as in sample teacher questions (p.89).
Systematic listing strategies	This may be implicit within the probability aspects of the MYP Mathematics Skills Framework. If added into the MYP Mathematics Skills Framework as an additional topic / skill, consideration of Cartesian products to list outcomes would be suitable for the middle years; more formal learning about permutations and combinations should be left until post-middle years.
Apply and use limits of accuracy / upper and lower bounds	Experts agreed this is a suitable topic for inclusion within the middle years.
Use scale factors, scale diagrams and map scales	This is implicit from PYP Shape and Space Phase 4 (p.91).
Investigate and calculate best buys / unit rates with fractions	This skill may be implicit within the MYP Mathematics Skills Framework guidance on ‘ratio and proportion’ (see section 1.5).
Compound units such as speed and rates	This is not stated explicitly, but links to the themes about curriculum guidance on ratio and proportion discussed in section 2.5.
Equation of a circle with centre at the origin	Experts agreed this is a suitable topic to include within the extended challenge level.
Constructions e.g. bisectors, perpendiculars	Experts commented that compass-and-ruler constructions can provide a platform for deductive geometry reasoning.
Loci	Suggestion from the experts that the specification of location, degrees of freedom and loci should be included in the standard and extended level. Loci specifically is suggested to support the appreciation of degrees of freedom.
Bearings	Specified in PYP Shape and Space Phase 4 (p.91).
Nets and construction of 3D shapes	Implicit from PYP Shape and Space Phase 4 (p.91).
Understand and use relative frequency	The collection of relative frequencies in an experiment is suggested as a priority for development from the experts.

Theme	Comment
Evaluate statistical reports in the media	IB MYP Mathematics does not specify this explicitly, however the combination of the MYP Mathematics Skills Framework guidance and the wider IB approaches to learning suggest it is possible to embed this theme within current teaching practice.
Explore variations of means and proportions of random samples drawn from the same populations	Experts commented that an understanding that samples drawn from the same population may vary is a fundamental idea in statistics. Suggestion that this may be a suitable topic to include in the MYP Mathematics Skills Framework.

2.3.4 Discrete Mathematics

In addition to the main research questions, the IB had asked NFER to consider whether discrete mathematics should be reintroduced as a fifth branch within the MYP Mathematics Skills Framework. The other systems within the comparison do not feature discrete mathematics. Expert opinion also recommended that it would not be sensible to reinstate this as an additional branch. Although some aspects of networks and path problems were considered likely to be interesting and accessible to middle years learners, and provide a different aspect to mathematical ideas, they are not vital to the middle years and can easily be picked up in courses later on if needed without having been a part of middle years learning. The two elements of discrete mathematics that were embedded into the other branches (*Sets and Venn diagrams* embedded into the *Number* branch and *Algorithms* embedded into *Algebra* branch) were considered appropriate to the middle years, although some experts felt *Sets and Venn diagrams* may be better situated in the *Statistics and probability* branch, where the underlying concepts may be helpful to solve probability and chance problems involving conditional probability.

2.3.5 Digital technologies

Within the IB MYP Mathematics Skills Framework, use of ICT is only mentioned within the *Statistics and probability* branch. Some of the other systems in the curriculum comparison make stronger reference to use of technology to support mathematical learning. For example, the Singapore curriculum references the use of ICT within a 'guided inquiry' approach to develop learning. Guidance for their 'geometry and measurement' strand indicates that learners should use Geometer's Sketch Pad (GSP) or other dynamic geometry software to explore constructions. Within the ACARA curriculum, many of the learning statements have additional guidance indicating how ICT may be used to support learning.

At a more holistic and innovative level, during our main expert panel there was an interesting discussion of currently understood potentials for digital technology to transform teaching and learning. There are many computer-based packages that learners can use to process results efficiently across a wide variety of mathematical areas, such as computer systems to process calculus results or data sets. With sufficient scaffolding and instruction learners have the potential use these packages to explore mathematical ideas that are currently beyond their understanding. As an example, learners who do not have any proficiency in solving quadratic equations could still consider how to form a quadratic equation to model a real-life problem, use a computer algebra package to solve the equation and then focus their learning onto interpreting the results. This perhaps opens up a far wider philosophical debate about what it means to be able to 'do' mathematics, and what skills are necessary, but it is certainly an interesting point to consider in a technologically advanced age. This lies well beyond the scope of this research project, but is something that the IB and mathematics teachers may wish to consider in the future.

2.4 Depth of learning

2.4.1 Challenge levels within IB MYP

In terms of depth of learning, research findings from both the curriculum comparison and expert panel work suggest that the topics and skills within the MYP Mathematics Skills Framework broadly mirror the levels of demand seen in other middle years curriculums and systems. There may be scope for the IB to reconsider the challenge levels of some topics, as detailed in Table 2.5. A commentary is provided for each topic which summarises key comments from both the expert panel and form NFER's perspective based on evidence from across NFER's research activities. More detailed commentary from the expert panel discussions can be found in Supplementary Report 1 – Expert panel high level report.

Table 2.5: Challenge levels within IB MYP

Branch	Topic	Current challenge level	Comment
Geometry and trigonometry	Similarity and congruence	E	Although the current MYP Mathematics Skills Framework skills are appropriate for the *Extended* challenge level, it seems surprising to have no reference to similarity within the *Standard and extended* level, especially as this links strongly to understanding of proportionality (in *Standard and extended* guidance for Number)
Number	Fractional Exponents	E	Fractional exponents appear in both the Algebra and Number branches but at different challenge levels. Having them based in the same challenge level might foster cross branch links.

Branch	Topic	Current challenge level	Comment
Number	Number bases	E	Suggestion from the expert practitioners that this often gets taught only as a standalone topic to fulfil the MYP Mathematics Skills Framework requirements. At the *Standard and extended* challenge level, this topic supports understanding of counting systems, but the teaching of operations using different bases seems unnecessary.
Statistics and probability	Standard Deviation	E	Suggestion from the experts and curriculum comparison that learners ought to learn to apply and interpret standard deviation at a basic level. Manual calculation could be retained at the Extended challenge level to help understand the underlying concepts. As standard deviation is included in the DP SL studies it would be useful for pupils to have this introduction to it in the middle years.

During both the curriculum comparison and the expert panels, three further issues emerged linked to depth of learning:

- Does the written guidance enable an appropriate conceptual understanding of the topic that can be developed?
- Does the curriculum structure support learners in making connection across different mathematical ideas?
- Are learners able to develop higher order thinking skills and be able to apply their knowledge to increasingly sophisticated settings?

Considering only the MYP Mathematics Skills Framework, the Phase 1 activities suggest there may be some concern over these issues. However, it must be noted that within IB MYP Mathematics, much of this 'richness' of learning is explicated within the full *MYP Mathematics Guide*. A key discussion point during the expert panel was the extent to which practitioners would be able to use the content in the MYP Mathematics Skills Framework to build their own 'scope and sequence' plans throughout the middle years to allow successful development of conceptual learning. Within other systems that adopt a year-by-year approach, there is a more clear progression through the topics implied by the curriculum structure. We understand that at a school level these ideas are promoted within other IB documents, such as *MYP: From principles into practice*, and are supported via professional development, but the structure of the MYP Mathematics Skills Framework itself does not contain the level of support and structure seen within other middle years systems.

The Phase 1 research activities also indicated an issue linked to depth of learning within the *Statistics and probability* branch – a more detailed analysis of this is presented in section 2.6.

The structure / layout of the current MYP Mathematics Skills Framework, presenting the content as a set of discrete topics, does not lend itself well to supporting making connections between branches, however this same criticism could also be made of other systems within this comparison. During the expert panel review, one expert commented that any approach to simply listing topics may preclude the richer connections between topics explicitly stated, and that there may be merit in rethinking the design of the MYP Mathematics Skills Framework (see section 2.2 for case studies of other systems / designs). As noted in section 2.2.2, the wording of the written guidance in other systems sometimes provides a greater sense of the inter-relationships between mathematical ideas.

The final aspect of depth of learning highlighted by the research activities carried out in Phase 1 was linked to higher order thinking within mathematics. The desk research carried out indicated that the majority of the other systems include not only problem-solving and reasoning skills, but also an additional element of meta-cognition; an ability to evaluate methods and results and consider improvements to strategies used. An emerging trend in other systems is to use mathematics as a modelling tool, and, as one expert stated, “to apply a mathematical lens to social problems”. The wider *MYP Mathematics Guide* certainly promotes much of this agenda through the use of global contexts and enquiry questions and in the MYP Principles to practice, approaches to teaching and learning; however there may be benefit from considering meta-cognition skills more explicitly in the MYP Mathematics Skills Framework itself given its importance in learning and development.

Case study: GAIMME Report

The GAIMME report was written targeting classroom teachers by COMAP (Consortium for Mathematics and its Applications) and SIAM (Society for Industrial and Applied Mathematics). It promotes the importance of mathematical modelling in mathematics classrooms.

The report aims to help teachers incorporate mathematical modelling in the classroom. They define mathematical modelling as ‘a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.’ (COMAP and SIAM, 2016). Mathematical modelling involves using mathematics to connect to and to solve real world problems. This fits with the overarching IB philosophy on promoting connections to global learning. The report provides concrete examples for different grade levels and abilities for practitioners. Useful classroom examples are outlined that can be easily adopted into the classroom and encourage teachers to ensure that their mathematics teaching is embedded into the real world. The report gives advice on the best ways to implement modelling into the classroom and how to create appropriate opportunities. There is advice on opportunities for assessment of learning when using modelling. The report also incorporates a teacher 'FAQ' section which has information on potential challenges and how to overcome them. The report is also supported by a range of reliable and robust research evidence.

The following sections examine two areas: (1) Ratio and proportion, and (2) Statistics and Probability as being the most variable within the curriculum comparison in terms of the ways in which different systems set out learning expectations and provide written guidance. Results from the expert panel are also explicated for each of these areas.

2.5 Ratio and proportion

2.5.1 Key issues

The curriculum comparison work carried out showed a wide variety in the supporting written guidance on ratio and proportion in each of the systems, as detailed in Table 2.6. The expert panel discussions noted that developing an understanding of the links between fractions, decimals, percentages, ratio and proportion and the use of multiplicative relationships should be a key feature of middle years mathematics learning.

2.5.2 Ratio, Proportion and Percentages – curriculum comparison

All systems, including the MYP Mathematics Skills Framework, specify sharing in a given ratio and use of direct and inverse proportion.

Table 2.6: Ratio and Proportion in different systems

Curriculum / System	Branch / Strand structure	Ratio and proportion written guidance	Percentages written guidance
IB MYP	Part of Number branch		
England Pearson GCSE National curriculum	'Ratio, Proportion and Rates of change' is a separate strand. Percentages occur both in	Written guidance includes the connections between graphical representations of proportional relationships and gradients of graphs as a rate of change. GCSE	Written guidance includes using percentages to make comparisons, percentage change and financial mathematics.

Curriculum / System	Branch / Strand structure	Ratio and proportion written guidance	Percentages written guidance
	the 'Number' and 'Ratio, Proportion and Rates' strand	subject guidance also states a requirement to relate ratios to fractions and linear functions.	
Cambridge iGCSE	Part of the 'Number' topic	iGCSE includes using common measures of rate and average speed.	Curriculum guidance for percentages centres more on skills e.g. calculating percentage increase of decrease. References are specified elsewhere relating to profit / loss and the compound interest formula.
US Common Core State Standards	'Ratio and proportional reasoning' forms its own domain	Written guidance emphasises the connections between multiplication and division as they relate to ratio and rate problems, and between ratios and fractions.	Percentage-based links are also explicitly stated in Grade 7 guidance, with a significant focus on financial contexts.
ACARA framework	Ratio and proportion are embedded within the 'Number and Algebra' content	Written guidance makes explicit links to the connections between rate and ratio problems and the use of fractions and percentages as part of efficient problem solving.	Money and financial mathematics are also specified, including the use of percentages (e.g. interest, profit / loss)
Singapore	'Ratio and proportion', 'Percentages' and 'Rate and Speed' are specified sub-sections within the 'Number and Algebra' content	The 'Learning Experiences' include links between fractions and ratios, and formulating linear equations. 'Rate and Speed' includes currency exchange rates, interest rates and tax rates.	'Percentages' includes developing an appreciation of how percentages are used in common financial situations e.g. bills/receipts.

Curriculum / System	Branch / Strand structure	Ratio and proportion written guidance	Percentages written guidance
Quebec Education Program	The approach is different to other systems. 'Understanding of proportionality' is one of the three themes (along with 'Number and operation sense' and 'Processes' that underpin the main topic areas in the curriculum.		

The curriculum comparison work suggests that the written curriculum guidance is considerably more comprehensive for ratio and proportion within the majority of other systems than it is within the IB MYP Mathematics Skills Framework. An important note, however, is that in many other systems, the curriculum is generally more prescriptive in terms of specifying what sorts of skills learners should be able to demonstrate. Within the context of ratio and proportion, the written guidance in other curriculums / systems is more explicit in terms of connecting ratio to fractions, algebraic relationships or making connections between proportionality and graphical representation. The idea of making connections between mathematical topics is clearly emphasised within the wider IB *MYP Mathematics Guide*, however in considering only the written guidance in the MYP Mathematics Skills Framework, some key conceptual learning could be in danger of being overlooked by MYP practitioners.

2.5.3 Expert panel comments

In preparation for the expert panel, experts were sent a questionnaire to complete. This questionnaire asked experts for their opinion on the current topics listed in the MYP Mathematics Skills Framework and asked two questions; A) whether they felt they were relevant to the middle years and B) if so at which challenge level? Experts were also offered the opportunity to suggest additional topics that they felt were necessary and to comment generally on the MYP Mathematics Skills Framework. The questionnaires indicated that the current MYP Mathematics Skills Framework does not offer as rich a source of written guidance within these topics as other systems. There was a strong feeling that the links between the concepts of ratio / proportion and scaling were not explicit in the IB MYP Mathematics Skills Framework, and as such the rich connection between the ideas of ratio / proportion and similarity within a geometrical context may not be sufficiently developed within IB MYP Mathematics. One expert commented that "In the case of ratio, there is no more important topic in applying mathematics to describe and understand the world."

The experts also agreed that similarity is a topic that can be readily understood by students even at the *Standard and extended* challenge level, providing scope for learners to think about shapes as a whole object, as opposed to only considering isolated properties such as side lengths or angles. In this sense, there is a rich link between the concepts of proportionality and transformation geometry / isomorphic objects that again is easy to overlook in the compartmentalised nature of the IB MYP Mathematics Skills Framework.

2.5.4 Literature review

Case study: The Nuffield Foundation

The Nuffield Foundation has produced interesting summaries of key ideas in teaching mathematics on a variety of topics. Within ratio and proportion, they state that "The core idea that underpins ratio and proportion is that any two numbers can be expressed as multiples of each other". Their research also suggests that proportional reasoning problems usually involve juggling four numbers, and that from a neuro-science perspective, early adolescence is a stage in which "learners become more able to make these compressions which chunk separate elements together". They assert that to generalize about ratio takes "several years and many experiences" and they offer useful guidance and resources to support the teaching and learning of proportionality, including whole school approaches.

2.6 Statistics and probability

2.6.1 Key issues

This branch was highlighted by the curriculum comparison work as being the most variable in terms of the ways in which different curriculums / systems set out learning expectations. This branch also offers significant scope for innovation, and the expert panel meetings indicated some potential for 'missed opportunities' within the MYP Mathematics Skills Framework.

2.6.2 Curriculum comparison

The overall content listed in the MYP Mathematics Skills Framework is broadly similar to other middle years programmes. The introductory text in this branch also indicates that learners should develop a wider appreciation of the power and limitations of statistics and describes some of the wider skills that should be developed. Table 2.7 summarises some interesting aspects provided within some of the other systems / curriculums.

Table 2.7: Statistics and probability – written guidance in some non-IB systems

	Data collection	Graphs, charts and diagrams	Measures of central tendency / spread	Probability
ACARA	Includes consideration of data sources, techniques, and consideration of everyday questions that can be addressed via	Guidance provides greater detail in places to support interpretation skills (e.g. use of skewed / symmetric / bi-modal). In Year 10, there is a	Investigate the effects of individual data values / outliers. Compare data displays using	

	Data collection	Graphs, charts and diagrams	Measures of central tendency / spread	Probability
	application of statistical techniques	reference to evaluating statistical reports in the media	location (centre) and spread	
Singapore	Includes consideration of appropriate statistical representation and justifying choices, and using data to make informed decisions, predictions and inferences.	Consider representations of data from newspapers and other sources and misleading representations	Consider how different measures of central tendency are affected by extreme values	Compare and discuss experimental and theoretical values using computer simulations

2.6.3 Expert panel comments

Experts were asked to review the *Statistics and Probability* branch and to provide feedback on how appropriate it was in terms of content, structure and fitness for purpose. The experts generally agreed that the statistics elements of this branch in the MYP Mathematics Skills Framework seemed to focus more on the procedural aspects of learning. One expert commented on the idea that when data is presented it forms a distribution that can be analysed, and that this does not come across strongly within the MYP Mathematics Skills Framework. Small changes to the wording in the guidance may improve this. There was also a rich discussion about whether the MYP Mathematics Skills Framework, in its current form, promotes the intrinsic links between statistics and probabilistic models, and the increasing opportunities to make use of technology to help learners develop a deeper understanding.

Case Study: ComputerBasedMath.org

CBM emphasises using maths for everyday living and using computers to help us do that. Their solution helix involves four equally weighted problem solving steps, define the question, translate from world into a mathematical question, compute the answer and interpret the results. This is different from traditional problem solving where over 80 per cent of the time is spent on the computing / calculating stage. Using computers means that scenarios are only constrained by the conceptual matter, making them more accessible to students. Since 2013 Estonia has been using this approach to build a new school statistics course (CBM, 2013). At present there are no evaluative studies of this project, however it is an interesting innovation in approaches to middle years mathematics.

2.7 The age continuum

To gain further insights into the links across the spectrum of IB programmes with regards to mathematical learning NFER compared first the prior learning topics stated in the DP programme guide, at both Standard Level and Higher Level, and then the PYP Mathematics scope and sequence to the MYP Mathematics Skills Framework. Gaps in pre-requisite knowledge and topical coverage differences are described in this section.

First impressions of the DP prior learning content showed a higher level of detail and prescription compared to that in the MYP Mathematics Skills Framework.

The topics listed in the DP pre-requisite lists for Standard Level that were not explicitly mentioned in the MYP Mathematics Skills Framework are:

- Mappings of the elements of one set to another
- Properties of order relations < >
- Compass directions
- Bearings

Bearings are listed explicitly within the PYP *Scope and Sequence* document. The other topics are not stated explicitly in the written guidance for PYP or MYP, but are likely to be implicit within the IB Mathematics guide as a whole beyond the MYP Mathematics Skills Framework.

Topics that were listed in the DP pre-requisite lists for higher level include both the above and additional topics:

- Factorization
- Completing the square
- Rationalizing the denominator

Although these skills are not explicitly stated within the MYP written guidance, they may be implied by the topics and skills in the current MYP Mathematics Skills Framework. NFER's examination of DP mathematics curriculum specifications noted that other content areas provided more detail than that covered in the content within the middle years. These areas may be covered by teachers that are aware of the DP curriculum but are not made explicit within the MYP Mathematics Skills Framework. For example, one area of prior learning expected is to be able to add and subtract algebraic fractions. In the MYP Mathematics Skills Framework, it is only suggested that students learn to solve equations involving algebraic fractions. Further areas that are not explicit within the MYP Mathematics Skills Framework, but appear as expectations in the prior learning topics for both DP Mathematics Standard Level and Mathematics Higher Level are:

- Appreciation of errors
- Rational coefficients
- Simultaneous equations in two variables

The expectation that students use ‘examples from other subject areas’ (p.15 of the *MYP Mathematics Guide*) could be argued to be implicit within the design of the *MYP Mathematics Guide* generally where inter-disciplinary learning is emphasised.

Further evaluation of the MYP to DP continuum, based on Phase 2 research activities, is covered in Section 3.

3 Phase 2 – Programme implementation

Chapter outline

This chapter presents the findings from Phase 2 of the research study, the online questionnaire and the follow up interviews with IB MYP Mathematics practitioners. Practitioners were asked for their opinions and experiences using the MYP Mathematics Skills Framework. Practitioners were also asked about their experiences of the IB continuum and the links between the PYP and DP programmes.

Sections 3.1 and 3.2 describe the research design and data analysis methodology for Phase 2. Section 3.3 presents the data in relation to each of the four key research questions within Phase 2.

The questionnaire was completed in full by 518 practitioners in 279 schools from the three IB regions: IB the Americas (IBA), IB Africa, Europe and the Middle East (IBAEM) and IB Asia-Pacific (IBAP). Follow up interviews were carried out with four teachers, two from IBA, one from IBAP and one from IBAEM.

3.1 Research Methodology

The overarching aim of the Phase 2 of this research project was to provide insights into practitioner views and experiences of using the MYP Mathematics Skills Framework in schools. The research activities of Phase 2 involved a practitioner questionnaire and practitioner interviews.

These activities aimed to address the following four research questions:

1. What are school perceptions of the MYP Mathematics Skills Framework?
2. How are schools and teachers using the MYP Mathematics Skills Framework in their planning?
3. What facilitates school success, or acts as stumbling blocks, when implementing the MYP Mathematics Skills Framework?
4. Are changes or refinements needed to aspects of the MYP Mathematics Skills Framework to maximise a successful Mathematics programme implementation?

3.1.1 Research Design

This research used a mixed methods approach. An online questionnaire and structured interviews were used to collect both quantitative and qualitative data. The use of an online questionnaire was chosen due to the benefits of anonymity and reach to the global audience needed for this survey (Hartas, 2010). Additionally an online questionnaire was deemed the most practical in the short time frame of the project. Structured interviews were used to gain more detailed data (Leonard, 2003) to build on the understanding gained from the initial questionnaire.

3.1.2 Data collection

The data was collected in two stages. The first stage used the online questionnaire to collect initial data from MYP mathematics teachers and heads of mathematics or STEM departments. Data was collected globally. The questionnaire asked teachers for their perceptions of the current MYP Mathematics Skills Framework, and mathematics within the MYP, through a range of questions. The second stage involved four teachers selected to participate in a structured follow up interview. Structured interviews were chosen to allow for both more reliable information to be gathered and for comparison between interviewees (Hartas, 2010). This allowed for more in depth analysis of teachers opinions. Descriptive statistics summarizing the data from the questionnaire are provided in Appendix B.

3.1.3 Research Instruments

An online questionnaire was created aimed at IB MYP mathematics practitioners. A follow up interview schedule was created based on findings from the questionnaire.

The Questionnaire

The questionnaire was designed to address the four research questions of Phase 2. It was offered to all IB schools that use MYP. It was available in English, French or Spanish.

The questionnaire was separated into four distinct sections.

- Section 1 collected data on predominantly demographic information such as the number of years of teaching experience and of the IB MYP system. This section also included questions about the participant's role in the school, and in particular whether they held a leadership position within a mathematics department. Participants were then routed to questions either about how useful they find the MYP Mathematics Skills Framework for co-ordinating planning across their department or to inform planning for their classes, depending on their role in the department. Section 1 contained 11 questions.
- Section 2 collected data on how teachers use the MYP Mathematics Guide in schools. Although the focus for this research project is on the MYP Mathematics Skills Framework, it was important to ask some high-level questions about other sections of the MYP Mathematics Guide to provide evidence on depth and breadth of MYP mathematics. Section 2 contained 25 questions, with an additional 11 if participants stated that they used PYP or DP written documents to support MYP planning.
- Section 3 explored practitioners opinions of the guidance provided in one of the four branches of the MYP Mathematics Skills Framework. Each participant was allocated a branch at random. In this section, participants were asked for

granular detail about which topics they include in their teaching, which topics they believe are relevant to middle years mathematics and also to rate the quality of the written guidance for the branch. Section 3 contained seven questions. Four of these required participants to rate each statement from the MYP Mathematics Skills Framework for their allocated branch. One open question was included asking about additional topics that may be suitable for MYP mathematics.

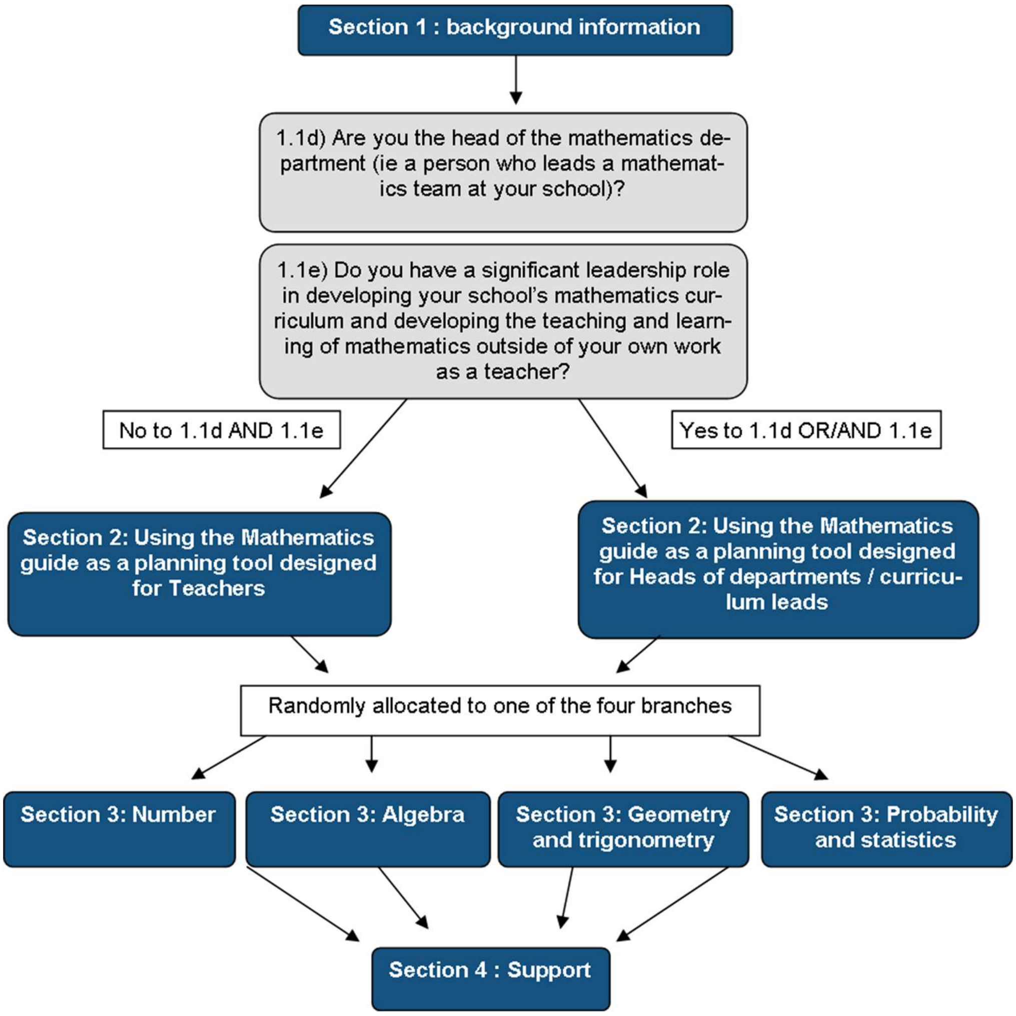
- Section 4 collected data about what support mechanisms practitioners use and how useful they found them. These support mechanisms ranged from professional development and on-line support provided by the IB to opportunities to work collaboratively within a school or a network of schools. Section 4 started with two questions about what support mechanisms were used, and ratings for these. Section 4 then contained a further ten questions about support from the IB and practicalities of delivering MYP mathematics.

To facilitate analysis, the majority of questions were closed questions using a 6 point rating scale. A 6 point scale was chosen as it requires a decision to be made rather than presenting a neutral option (Cohen *et al.*, 2007). For all questions, a rating of 1 indicated the most positive response (e.g. 'strongly agree' or 'highly relevant') and a rating of 6 indicated the most negative response (e.g. 'strongly disagree' or 'not at all relevant'). As well as allowing analysis of individual frequencies, this system also allows analysis of levels of agreement / disagreement:

- from the 6 point scale we can infer that a rating of 1,2 or 3 indicates some form of agreement and 4,5 6 indicate some form of disagreement
- from the 6 point scale we can infer that a rating of 1 or 2 indicates a stronger level of agreement and 5 or 6 indicate a stronger level of disagreement.

The pathways through the questionnaire are illustrated in Figure 2.

Figure 2 Pathway through the questionnaire



Questionnaire distribution

In total 1,417 schools were contacted about participating in the questionnaire. As per the IB protocol, the MYP co-ordinator was contacted. They were then asked to forward on the information to the head of department / mathematics lead in their school and up to four teachers. Each of these individuals was given a unique login for the questionnaire. The head of department logins allowed for equal distribution of heads of department to a branch of the MYP Mathematics Skills Framework. This allocation also took account of their region. For example the first head of department that filled in the survey from IB Americas was allocated the number branch, the second from IB Americas would then be allocated the algebra branch and so on. The MYP co-ordinator could also request additional login details for additional teachers. The questionnaires were distributed in this way to help ensure the representativeness of the results and to minimise regional bias. A total of 679 responses from 279 different schools were received. Of these, 518 were fully completed responses were received and 161 were partially completed. The breakdowns of the respondents are shown in Table 3.1 and Table 3.2. In order to still use the data from partial responses, we have completed the analysis of each question including all respondents. It is important therefore to consider the sample numbers, N, for each question which is reported underneath to gain a clear understanding.

Table 3.1: Sample by role

Role	N
Head of mathematics department	361
Teacher	276
No response	42
Total	**679**

There were some discrepancies between the regions the school was registered to and the responses to the question in the questionnaire asking for the region they were based in. In analysing the data, the questionnaire responses were taken as an indication of region. For the participants that did not respond to this question, the region the school was registered to was used.

Table 3.2: Sample by region

Region	N
IBA	358
IBAEM	179
IBAP	142
Total	**679**

The interview schedule

The penultimate question of the questionnaire asked respondents whether they would be willing to take part in a follow up interview. Four of these respondents were approached for a follow up interview. To ensure that the follow up interviews were representative respondents were chosen from each region. The interviewees were chosen as having a range of responses to key questions identified in section two of the questionnaire. They were also selected based on whether they used the Primary Years Programme and Diploma Programme documentation in their planning. This allowed for questions to be asked about the IB continuum and so contribute to research question 2. This also enabled the interviews to gain a range of opinions.

A structured interview schedule was developed based on key areas identified in the questionnaire. The interviews aimed to gain further insights into the findings from the questionnaire data and to provide more in depth qualitative data to address the research questions.

The interviews were carried out online using Skype. They lasted a maximum of 20 minutes and involved a set of six structured questions. Two researchers carried out each interview to ensure reliability of the findings. During the interview detailed notes were taken and then shared for cross validation. Immediately after the interview notes were discussed and reflected on to ascertain the two researchers' views and findings from the interview. This discussion helped to establish a consistent understanding of the interview and resolve any discrepancies.

3.2 Data analysis

The quantitative questionnaire data was analysed using SPSS statistical software. Descriptive statistics and statistical tests were carried out to identify areas of the data to report on. The qualitative questionnaire data was analysed by coding the recurring themes. Any responses in French or Spanish had to first be translated to English. The qualitative data from the structured interviews was analysed using key theme analysis considering the notes from each interviewee for each question.

3.3 Research findings

Practitioners answered different sections of the survey depending on whether they categorised themselves as being a head of department or having a significant role in planning in mathematics or whether they did not. These two categories will be referred to throughout the analysis as HoD (head of department) and Teacher.

3.3.1 What are school perceptions of the MYP Mathematics Skills Framework?

Throughout the questionnaire practitioners were asked to rate their agreement with statements on a scale of 1 to 6 with 1 being strongly agree and 6 being strongly disagree. Practitioners were asked for their views on the overall suitability of the MYP Mathematics Skills Framework content, specifically considering how the MYP Mathematics Skills Framework supports future learning. Their average score is reported in Table 3.3. Both HoDs and Teachers rated the statements similarly, with teachers rating slightly stronger agreement (in line with the rating system used, a lower mean score indicates a higher average level of agreement). Some positive comments were made during teacher interviews that the MYP Mathematics Skills Framework is student friendly and works well alongside state requirements.

Table 3.3: Practitioners' perceptions of the overall suitability of the framework content

	HoD N	HoD average (M)	Teacher N	Teacher average (M)
MYP mathematics prepares students well for external assessments within the middle years	329	2.98	250	2.80
MYP mathematics prepares learners well for further study	329	2.49	249	2.34
Overall the content provides learners with sufficient mathematical knowledge for future learning in general	232	2.33	244	2.31

Practitioners were also asked to consider the suitability of the MYP Mathematics Skills Framework in terms of preparing learners for the IB DP programme. Ratings of 1, 2 or 3 were treated as positive ratings and ratings of 4, 5 or 6 were treated as negative ratings.

Generally practitioners felt that the content indicated in both *Standard and extended* challenge level guidance and the *Extended* challenge level guidance prepares learners well for DP study. Both HoDs and Teachers were slightly more positive about the *Standard and extended* content (68% positive agreement for both HoDs and Teachers) than the *Extended* content (58% and 60% positive agreement for Teachers and HoDs respectively).

There was no significant difference shown between practitioners' experience and how they rated the suitability of the overall content. For Teachers the general trend showed that as their years of experience increased, their ratings became slightly more negative, but this difference was minimal and the rating remained positive.

One of the interviewees who uses the DP documentation in their planning did comment that the link does not always work and that the flexibility of the MYP programme sometimes does not link smoothly with the level of prescription in the DP programme.

Breadth of learning by branch

Practitioners were asked to rate their allocated branch on a scale of 1 to 6 in terms of how well it provides sufficient breadth of learning at a holistic level. A rating of 1 meant that they strongly agreed that the branch provided sufficient breadth and a rating of 6 meant that they felt it provided insufficient breadth.

There were no significant differences found between HoD and Teacher perceptions for breadth of learning. Table 3.4 shows the combined ratings for HoDs and Teachers. The largest variance was for the *Number* branch with the mean rating from HoD of 2.47 and the mean Teacher rating at 2.83. However, this still shows agreement from both that the breadth of learning is generally appropriate. Geometry and trigonometry was the only branch that received any responses at rating 6 indicating insufficient breadth, however this represents a very small proportion of respondents.

Table 3.4: Overall ratings of breadth of learning per branch

Rating	Number (%)	Algebra (%)	Geometry and trigonometry (%)	Statistics and probability (%)
1	12.3	17.6	17.3	10
2	44.2	50.7	35.3	42.1
3	22.5	21.1	31.7	35.0
4	10.9	7	12.2	8.6
5	10.1	3.5	2.9	4.3
6	-	-	0.7	-

Practitioners were also asked to rate on a scale of 1 to 6 whether each topic was highly appropriate for inclusion in the MYP Mathematics Skills Framework or not appropriate at all. Generally practitioners were positive, with no average ratings below a rating of 3, indicating that respondents generally agreed that all topics within the current MYP Mathematics Skills Framework are appropriate within the middle years. Table 3.5 indicates topics that stood out as having lower ratings; all of these are from the *Extended* challenge level.

Table 3.5: How appropriate is a topic for inclusion within the framework

Branch	Topic	Mean rating	Challenge level
Number	Fractional Exponents	2.08	E
Number	Logarithms	2.31	E
Number	Number Bases	2.26	E
Algebra	Logarithms with different base number	2.38	E
Algebra	Functions and graphs	2.12	E
Geometry and trigonometry	3 dimensional co-ordinate geometry	2.38	E
Statistics and probability	Standard Deviation	2.56	E
Statistics and probability	Measures of Dispersion	2.33	E

Statistics and probability	Conditional Probability	2.31	E

Interview responses supported this theme emerging from the questionnaire, with interviewees suggesting that they often had the most difficulty with the *Extended* challenge level content. Two interviewees identified this as an area that often had the most gaps in terms of progression to higher level DP courses. Another commented that teachers struggle with the *Extended* challenge level and how to incorporate it into their teaching plans.

This section of the questionnaire also gave respondents an opportunity to suggest any topics they felt were missing from the current MYP Mathematics Skills Framework For each branch the most commonly cited topic areas are listed below.

Number

- Surds
- Irrational numbers
- Algebra
- Polynomials

Geometry and trigonometry

- Polar coordinates
- Circle geometry
- Radian angle measures
- Trigonometric graphs

Statistics and probability

- Normal and binominal distribution
- Planning statistical analysis
- Combination and permutation

Depth of learning by branch

The survey design meant that each respondent was randomly allocated to one of the four branches of the MYP Mathematics Skills Framework to answer in depth questions about the granular detail. Practitioners were asked to rate their allocated branch on a scale of 1 to 6 in terms of how well it provides sufficient depth of learning. A rating of 1 meant that they strongly agreed that the branch provided sufficient depth and a rating of 6 meant that they felt it provided insufficient depth.

There were no significant differences found between HoD and Teacher perceptions of depth of learning. The largest variance was again for the Number branch with the mean of HoD responses at 2.31 and the mean of Teacher responses at 2.53. However, this is small and still demonstrates a high agreement from both groups with the statement. The Statistics and probability branch had the largest variance in

Teacher and HoD positivity ratings, with 10 per cent more HoDs rating the branch positively for depth of learning than Teachers.

Table 3.6 shows combined HoD and Teacher ratings for breadth of learning. The majority of practitioners considered the depth to be appropriate, with only a small proportion of respondents considering the MYP Mathematics Skills Framework to provide insufficient depth.

Table 3.6: Overall rating of depth of learning by branch

Rating	Number (%)	Algebra (%)	Geometry and trigonometry (%)	Statistics and probability (%)
1	17.6	22.6	15.6	13.0
2	44.9	43.8	42.2	39.9
3	22.8	26.3	26.7	36.2
4	9.6	7.3	12.6	6.5
5	4.4	-	3.0	2.2
6	0.7	-	-	2.2

3.3.2 How are schools and teachers using the MYP Mathematics Skills Framework in their planning?

Practitioners were asked to rate their agreement with the statement 'the mathematics guide in its current form is a useful planning tool' with 1 being strongly agree and 6 being strongly disagree.

Table 3.7: Percentage agreement with how useful the current Mathematics Guide is as a planning tool

Rating	HoD (%)	Teacher (%)
1	22.9	22.7
2	29.9	28.6
3	24.6	28.2
4	10	11.8
5	9.7	6.7
6	2.9	2
Mean	**2.62**	**2.57**

Practitioners were asked to consider how each branch within the MYP Mathematics Skills Framework supports their planning. The breakdown of ratings is displayed in Table 3.8. Interview responses were mixed about how practitioners feel about the current MYP Mathematics Skills Framework as a planning tool. Although some interviewees acknowledged the importance of the flexibility for a global framework, others identified this as a main difficulty. Some suggested that the MYP Mathematics Skills Framework was too brief which meant that when planning teachers, need to rely on their own experience or additional tools which then can create inconsistencies between classes and schools.

Table 3.8: Planning support by branch

		Number		Algebra		Geometry and trigonometry		Statistics and probability	
		HoD	Teacher	HoD	Teacher	HoD	Teacher	HoD	Teacher
Rating	N	326	248	326	249	323	245	323	245
	1	17.0	15.3	17.8	17.7	18.6	15.9	18.3	13.9
	2	38.7	40.7	39	36.5	36.2	38	36.5	39.2
	3	21.5	26.2	21.5	27.7	24.5	29.8	24.8	30.2
	4	14.4	11.3	13.8	12.0	12.7	9.8	12.4	10.6
	5	6.1	5.6	6.1	5.2	7.1	4.9	7.7	4.5
	6	1.8	0.8	1.8	0.8	0.9	1.6	0.3	1.6
Mean		**2.59**	**2.54**	**2.57**	**2.53**	**2.56**	**2.55**	**2.56**	**2.58**

Across all four branches there was no significant difference between HoD and Teacher perceptions of whether the branches provide enough information for effective learning. There was no significant difference between the four branches, with all four branches rated similarly.

Although very few respondents gave the lowest possible rating of 6, approximately 20 per cent gave a negative rating, i.e. approximately 1 in 5 participants felt the topics / skills stated did not provide sufficient information to help them plan. In each branch, a slightly higher proportion of HoDs than Teachers gave negative ratings. It should be noted that at HoD level, this question was phrased as considering the topics in relation to 'your work in leading mathematics planning in your school', whereas Teachers were asked to focus on their individual planning.

Table 3.9 Positive / negative rating summaries

		Number		Algebra		Geometry and trigonometry		Statistics and probability	
		HoD	Teacher	HoD	Teacher	HoD	Teacher	HoD	Teacher
Rating	N	326	248	326	249	323	245	323	245
	Positive (1, 2, or 3)	77.7	82.3	78.3	82.0	79.3	83.7	79.6	83.3
	Negative (4, 5, or 6)	22.3	17.7	21.7	18.0	20.7	16.3	20.4	16.7

Our Phase 1 (curriculum comparison and expert panels) research activities suggested that some aspects of breadth and depth of learning in mathematics lay beyond the MYP Mathematics Skills Framework itself, and were intrinsically linked to the wider *MYP Mathematics Guide* and MYP philosophies of learning. As such, all practitioners were asked some more general questions.

Practitioners were asked about how to embed the IB philosophies of learning into their planning and teaching. They were asked to rate on a scale of 1 to 6 their agreement with the statement ‘it is easy to embed the topics and skills into the wider IB MYP philosophies of learning (as specified in the full *Mathematics Guide*)’, reported in Table 3.10.

Table 3.10: Responses to ‘it is easy to embed the topics and skills into the wider IB MYP philosophies of learning (as specified in the full *Mathematics Guide*)’

Rating	HoD (%)	Teacher (%)
1	11.1	9.8
2	32.6	31
3	32.0	35.9
4	15.1	13.9
5	6.5	7.8
6	2.8	1.6
N	**325**	**245**
Mean	**2.82**	**2.84**

Generally practitioners neither strongly agreed nor strongly disagreed that it was easy to embed the topics and skills into the wider IB philosophies, with mean ratings of 2.82 and 2.84 for HoDs and Teachers respectively. However, nearly one quarter of both HoDs and Teachers responded negatively to this question. This suggests that being able to embed the content within the MYP Mathematics Skills Framework into the wider IB philosophies for teaching and learning is a challenge for a proportion of practitioners.

Horizontal planning

Practitioners were asked for their views of how the MYP Mathematics Skills Framework supports them in horizontal planning. In *MYP: From principles into practice*, horizontal planning encompasses collaborative planning and teaching, with practitioners of the same year level working together between and within subject groups to plan the scope of learning for a particular year. Within the scope of this research project, the focus was placed on mathematics teachers being able to use the MYP Mathematics Skills Framework to plan effectively for a particular year group and create links between learning in each of the four branches.

Table 3.11 The framework allows me to identify appropriate subject content for each year group

Rating	HoD (%)	Teacher (%)
1	20.9	18.1
2	38.8	44.2
3	20.9	25.3
4	12	7.6
5	4.9	4.0
6	2.5	0.8
N	**252**	**198**
Mean	**1.95**	**2.04**

Nearly 20 per cent of HoDs responded negatively to this question (a rating of 4, 5 or 6) compared to just 12 per cent of Teachers. This suggests that perhaps this is an issue that is more relevant to HoDs as planning across years is a greater requirement for their role than it might be for Teachers.

All of the interviewees acknowledged that the MYP Mathematics Skills Framework currently does not provide much advice on how to identify appropriate subject content for each year group. Interviewees felt that this was an area that the MYP Mathematics Skills Framework could be improved in with more specific year by year expectations being helpful. Some felt that this lack of detail was sometimes frustrating and led to practitioners having to rely on their prior experience or other teaching schemes of work. One interviewee however did comment that the lack of detail in the MYP Mathematics Skills Framework was good as it allowed a level of flexibility, but it did mean that they had to spend time developing and adapting their own five year teaching plan for the middle years.

Table 3.12: When planning it is easy to make links between topics and skills specified in different branches

Rating	HoD (%)	Teacher (%)
1	14.8	13.1
2	41.5	35.1
3	23.4	29.0
4	13.2	16.7
5	6.5	4.5
6	0.6	1.6
N	**325**	**245**
Mean	**2.57**	**2.69**

For both questions there were no significant differences between HoD and Teacher perceptions. Just over 20 per cent of both HoDs and Teachers responded negatively, although only just over 7 per cent gave a rating of 5 or 6. This suggests that a small but significant proportion of practitioners do not always find it easy to make links between the topics and skills in different branches.

Vertical planning

In *MYP: From principles into practice*, the goal of vertical planning is to ensure continuity and progression from year 1 to year 5 and beyond. Within the scope of this research project, the focus was placed on mathematics teachers being able to use the MYP Mathematics Skills Framework effectively to plan for progression in learning across the Middle Years Programme age continuum. Practitioners' responses are reported in Table 3.13.

Table 3.13 The framework allows me to plan for effective year on year progression

Rating	HoD (%)	Teacher (%)
1	19.8	18.1
2	34.9	37.3
3	26.5	28.5
4	11.7	9.6
5	4.6	5.6
6	2.5	0.8
N	**324**	**249**
Mean	**2.54**	**2.5**

There was little difference between HoD and Teacher perceptions, with the majority of both groups reporting positively on how the MYP Mathematics Skills Framework helps them in vertical planning. A higher proportion of HoDs responded negatively (19%), again with just over 7 per cent giving a rating of 5 or 6. This suggests that at the more strategic planning level undertaken by HoDs, nearly 1 in 5 participants felt the current MYP Mathematics Skills Framework does not allow them to plan effectively year on year, with a small proportion finding this a real difficulty.

In terms of planning for progression one interviewee commented that this is difficult and requires additional time by planning leads to develop a programme for progression specifically for their school.

Practitioners were also asked whether they use the PYP and DP documentation in their planning. Table 3.14 shows their responses. HoD are more likely than Teachers to use the additional documentation to inform their planning and both are more likely to use the DP documentation than the PYP documentation.

Table 3.14 Percentage of teachers that use PYP and DP to help plan their courses

	HoD (%)		Teacher (%)	
	Yes	No	Yes	No
Primary Years Programme Scope and Sequence	18.5	81.5	11.9	88.1
Diploma Programme Mathematics Guide	56.7	43.3	42.6	57.4

Responses from both the questionnaire and interviews highlighted that the school context is a major factor as to whether PYP or DP written documents are used to support planning. If a school covers the full IB age continuum; they were much more likely to use the other programme documentation to support their planning. This finding was supported in the interviews with interviewees that work in schools covering more than one IB age range appearing to have a more evolved and developed continuum programme.

3.3.3 What facilitates school success, or acts as stumbling blocks, when implementing the MYP Mathematics Skills Framework?

Support mechanisms

In section four of the questionnaire practitioners were asked about the support mechanisms that are in place from the IB and how these support them in to implement the MYP Mathematics Skills Framework in their school.

Firstly practitioners were asked whether they had used any of the six chosen support mechanisms, then if they had said they had used a mechanism they were asked to rate how useful they found it. Table 3.15 and Table 3.16 show these responses.

Overall HoDs were more likely report using support mechanisms than Teachers. The IB's Online Curriculum Centre (OCC) forums and IB Professional Development (PD) events were cited as the two most used support mechanisms by both HoD and Teachers.

Table 3.15 Which of the following support mechanisms have you used in the last 2 years?

	HoD (%)		Teacher (%)	
	Yes	No	Yes	No
OCC (IB's Online Curriculum Centre)	88.4	11.6	81.2	18.8
Part of an IB network of schools	51.2	48.8	38.8	61.2
IB Answers	38.9	61.1	30.4	69.6
Online support from IB	38.0	62.0	30.8	69.2
Online networks e.g. social media groups	38.3	61.7	27.7	72.3
IB Professional Development (PD) events	82.8	17.2	68.3	31.7

Table 3.16: Practitioners ratings of support mechanisms usefulness

	Rating (%)						N
Support mechanism	1 (very useful)	2	3	4	5	6 (not useful)	
OCC (IB's Online curriculum centre)	29.0	30.7	22.0	8.9	8.5	0.9	449
Part of an IB network of schools	27.2	38.5	20.9	7.9	3.8	1.7	239
IB Answers	29.8	32	24.9	7.2	4.4	1.7	181
Online support from IB	27.9	33	22.3	7.8	5.0	3.9	179
Online networks e.g. social media groups	26.7	45.5	22.2	4.0	1.1	0.6	176
IB Professional Development (PD) events	33.9	39.7	19.7	3	3.7	-	401

On balance HoDs and Teachers indicated similar levels of agreement. For ratings of IB Answers and IB Professional Development events Teachers were very slightly more negative with an average rating -0.16 and -0.14 lower than the HoDs ratings.

There is some variance in the number of practitioners that answered each of these questions as they were routed depending on which support mechanisms they had reported having used.

All participants were asked about the quality of support provided by the IB. Table 3.17 indicates quite high variability in the responses to this question.

Table 3.17: IB provides useful support

Rating	HoD (%)	Teacher (%)	Overall (%)
1	12.3	11.6	12
2	27.3	23.2	25.6
3	26	26.3	26.1
4	12.7	13.8	13.2
5	10	9.8	9.9
6	5.3	4.9	5.2
Mean	**3.22**	**3.43**	

Practitioners that had taught in the IB system for less than one year were slightly more positive about the support that IB provides, with a mean rating of 2.72 compared to those practitioners who had taught for one to three years with a mean rating of 3.01. This is shown in Table 3.18.

Table 3.18: IB provides useful support – by experience

Years experience	N	Mean
Less than 1 year	57	2.72
1 – 3 years	183	3.01
4 – 9 years	188	3.02
10 – 19 years	52	3.08
20+ years	1	2

Potential stumbling blocks

In section four of the questionnaire practitioners were asked to rate their agreement with the following two statements:

- there is enough time to work collaboratively in the maths department to develop MYP learning opportunities
- there is enough time to work collaboratively with other departments to develop MYP learning opportunities.

The aim of these questions was to determine whether practitioners felt they had enough time to work on planning not only within their department but also across departments to provide interdisciplinary learning opportunities.

Table 3.19: Potential stumbling blocks – time / opportunities

	Rating (%)						N
	1 (agree strongly)	2	3	4	5	6 (disagree strongly)	
There is enough time to work collaboratively in the maths department to develop MYP learning opportunities	15.5	28.9	19.2	14.9	12.5	9.0	522
There is enough time to work collaboratively with other departments to develop MYP learning opportunities	10.3	19.5	20.7	17	20.7	11.7	522

Similar themes were found in the interviews with interviewees all valuing the importance of both interdisciplinary learning and collaboration in planning, but that in practice it depended on the amount of time both they and their colleagues had available.

Practitioners were also asked about how well they felt the IB philosophies worked in practice in their school and the emphasis that their school places on them. Practitioners responded positively with 43 per cent strongly agreeing that they valued the philosophies for learning. This positivity towards the IB philosophies was echoed in the practitioner interviews. In the questionnaire, a slightly lower percentage strongly agreed that developing the philosophies was a key priority for their team in their school (28%). This is shown in Table 3.20.

Table 3.20: Potential Stumbling blocks – IB philosophies for learning

	Rating (%)						N
	1 (agree strongly)	2	3	4	5	6 (disagree strongly)	
I value the IB philosophies for learning	43.2	36.9	15.2	3.6	1.0	0.2	521
Developing the IB philosophies in my school is a key priority for the mathematics team in my school	28.2	33.2	23.0	9.0	4.4	2.1	521

Collectively, Table 3.19 and Table 3.20 indicate that whilst participants value the IB philosophies and that at a whole-school level these philosophies are important, in reality practitioners report that there is not sufficient time to work collaboratively.

3.3.4 Are changes or refinements needed to aspects of the MYP Mathematics Skills Framework to maximise a successful Mathematics programme implementation?

In the survey, respondents had the opportunity to answer two open questions. The first was to suggest any topics missing from the branch they were answering about. The second was for any final comments.

This qualitative data was analysed by coding the key themes emerging from the responses. Any responses in French or Spanish were first translated to English before analysis. The main themes drawn out of the analysis are outlined in Table 3.21 with the number of times that theme was mentioned.

Table 3.21: Key recurring comments about the framework

Theme	Number of comments
More examples needed	29
More comprehensive guidance needed	19
MYP e-Assessment	18
Conflicts between IB and state / national requirements	13
Positive comment about MYP generally	5
Issues with links to DP and further assessment	5

Source: NFER (2016)

In terms of examples and guidance, in section 2 of the questionnaire, participants had been asked to rate how useful they find some aspects of the *MYP Mathematics Guide* to support planning. Tables 3.22 and 3.23 show the three questions from the practitioner questionnaire that gained the lowest ratings. This does suggest that for some practitioners there is a stumbling block in being able to use the exemplar material within the *MYP Mathematics Guide* to structure their own planning.

Table 3.22: Mathematics Guide – descriptive statistics for ‘general planning’ questions

Question	N	Mean	Standard deviation
Q2_2D The ‘Planning the mathematics curriculum’ examples on pages 16 and 17 provide sufficient support to allow me to plan other similar cross-year learning programmes	592	3.03	1.280
Q2_2E The ‘Statements of inquiry’ and ‘Inquiry questions’ examples on pages 20 to 23 provide sufficient support to allow me to develop my own examples.	593	2.96	1.345
Q2_2G The guidance in the Mathematics Guide allows me to plan appropriate courses that cater for students of different ability levels.	580	3.01	1.254

Source: NFER (2016)

Table 3.23: Mathematics Guide – rating analysis for ‘general planning’ questions

	Rating (%)						Cumulative Rating (%)			
Question[8]	1 Agree strongly	2	3	4	5	6 Disagree strongly	(1-3) Agree	(4-6) Disagree	(1&2) Agree	(5&6) Disagree
Q2_2D	9.6	28.0	31.8	15.2	11.3	4.1	69.4	30.6	76.6	15.4
Q2_2E	12.0	30.0	29.3	12.6	10.8	5.2	71.3	28.7	42.0	16.0
Q2_2G	9.1	29.8	29.5	17.9	10.0	3.6	68.4	31.6	38.9	13.6

3.3.5 Regional differences

All questions were analysed by region to assess whether there were any significant regional differences in practitioners’ answers. Participant responses from three regions IB Africa, Europe and the Middle East (IBAEM) IB Americas (IBA) and IB Asia-Pacific (IBAP) were compared. Analysis of Variance (ANOVA) tests were carried out to identify if there were any significant differences between the group means. Only the questions that had significant regional differences are reported here. For each, the number of responses, mean rating per region and standard deviation per region are tabulated. Each rating was on a 1 – 6 scale, with 1 indicating the most positive response and 6 the most negative response; as such, the lower the mean, the more positive the collective response was. The ANOVA analysis is provided in Appendix B.

[8] See Table 3.22 for the statement referred to by each question code.

School perceptions of the IB MYP Mathematics Skills framework

The one-way ANOVA showed that there was a significant effect of regional difference in how participants responded to questions about their perceptions of the overall MYP Mathematics Skills Framework for future study. Participants were asked to rate agreement on a scale of 1 to 6 for the following two statements;

- IB MYP mathematics prepares students well for external assessments within the middle years (e.g. statutory state / national tests)
- IB MYP mathematics prepares learners well for further study

Both questions had significant regional differences with respondents from IBA responding more positively than respondents from both IBAP and IBAEM. The number of responses and mean ratings per region for each statement are shown in Tables 3.23 and 3.24

Table 3.24: IB MYP mathematics prepares students well for external assessments within the middle years

	IBA	IBAEM	IBAP
N	167	85	77
Mean rating	2.80	3.15	3.18
Standard deviation	1.286	1.316	1.345

Table 3.25: IB MYP mathematics prepares learners well for further study

	IBA	IBAEM	IBAP
N	166	85	78
Mean rating	2.32	2.88	2.42
Standard deviation	1.199	1.416	1.251

How are schools and teachers using the MYP Mathematics skills framework for planning?

To gain an understanding of how practitioners use the MYP Mathematics Skills Framework for planning, participants were asked to rate their agreement on a scale of 1 to 6 to the following statements:

- The topics and skills in the *Number* branch provide me with enough information to plan for effective mathematics learning
- The topics and skills in the *Algebra* branch provide me with enough information to plan for effective mathematics learning
- The topics and skills in the *Geometry and trigonometry* branch provide me with enough information to plan for effective mathematics learning
- The topics and skills in the *Statistics and probability* branch provide me with enough information to plan for effective mathematics learning.

In terms of regional differences by branch, the one-way ANOVA showed some significant effects. Practitioners from IBAP responded more positively than practitioners from both IBA and IBAEM for *Number* and for *Algebra*. Practitioners from IBAP responded more positively than practitioners from IBAEM for *Statistics and probability.* There was no statistical significance in the regional differences for the *Geometry and trigonometry*. Table 3.25 provides summary statistics of the ratings by region for each branch of the Mathematics Skills Framework.

Table 3.25: IB MYP mathematics prepares learners well for further study

		IBA	IBAEM	IBAP
Number	N	290	157	127
	Mean rating	2.59	2.76	2.26
	Standard deviation	1.156	1.216	1.078
Algebra	N	290	157	128
	Mean rating	2.57	2.71	2.32
	Standard deviation	1.160	1.230	1.101
Geometry and trigonometry	N	284	157	127
	Mean rating	2.56	2.67	2.42
	Standard deviation	1.128	1.206	1.165
Statistics and probability	N	284	157	127
	Mean rating	2.56	2.74	2.36
	Standard deviation	1.090	1.204	1.132

Participants were also asked to rate agreement to the following statements on a scale of 1 to 6, with 1 being strongly agree and 6 being strongly disagree.

- The MYP Mathematics Skills Framework allows me to identify appropriate subject content for each year group
- When planning it is easy to make links between topics and skills in different branches

The one-way ANOVA showed that there was a significant effect of regional difference in how participants responded to the extent to which the MYP Mathematics Skills Framework allows them to identify appropriate subject content for each year group, with respondents from IBAP responding more positively than respondents from IBA, as shown in Table 3.26

Table 3.26: The framework allows me to identify appropriate subject content for each year group

	IBA	IBAEM	IBAP
N	290	156	128
Mean rating	2.56	2.49	2.11
Standard deviation	1.185	1.133	1.037

The analysis also showed that there was a significant effect of regional difference in participants' responses to how easy it is to make links between topics and skills in different branches when planning, with respondents from IBAP responding more positively than respondents from both IBA and IBAEM, as shown in Table 3.27

Table 3.27: When planning it is easy to make links between topics and skills in different branches

	IBA	IBAEM	IBAP
N	286	157	127
Mean rating	2.68	2.73	2.37
Standard deviation	1.161	1.113	1.045

Year-on-year progression

Participants were asked to rate how well they agreed with the statement 'the MYP Mathematics Skills Framework allows me to plan for effective year on year progression'. This was to gain an understanding of how participants felt the MYP Mathematics Skills Framework supported them in vertical planning. The one-way ANOVA showed that there was a significant effect of regional difference in how participants responded, with respondents from IBAP responding more positively than respondents from both IBA and IBAEM, as shown in Table 3.28 This suggests that respondents in the IBAP region were significantly more positive about how well the MYP Mathematics Skills Framework supports them in vertical planning than those respondents from the other two regions.

Table 3.28: The framework allows me to plan for effective year on year progression

	IBA	IBAEM	IBAP
N	289	156	128
Mean rating	2.57	2.64	2.26
Standard deviation	1.185	1.175	1.081

How schools and teachers perceive IB Support

In section four of the questionnaire, practitioners were asked to rate their agreement on a scale of 1 to 6 with the following statement 'IB provides useful support'. A rating of 1 meant they strongly agreed with the statement and a rating of 6 meant they strongly disagreed. The one-way ANOVA showed that there was a significant effect of regional difference participants’ responded to how useful the support IB provide is, with respondents from IBAEM responding more negatively than respondents from both IBA and IBAP, as shown in Table 3.29

Table 3.29: IB provides useful support

	IBA	IBAEM	IBAP
N	233	138	111
Mean rating	2.87	3.32	2.83
Standard deviation	1.388	1.430	1.198

4 Key findings and recommendations

4.1 The structure of the MYP Mathematics Skills Framework

4.1.1 The current structure – branches, topics and skills

The IB MYP Mathematics Skills Framework is broadly fit for purpose in its current form, however we would recommend some revisions to the current topics and skills, as detailed in Section 4.2 and there may be merit in considering alternative formats for the subject content specified. In terms of breadth and depth of learning, the *Standard and extended* challenge level content is broadly aligned to other middle years systems, and the majority of the content listed in the Mathematics Skills Framework seems appropriate to the needs of middle years learners. The *Extended* challenge level includes a wider range of content than other middle years systems, some of which was not considered appropriate for middle years learners by experts or a significant proportion of MYP mathematics teachers. In this sense, the *Extended* challenge level content may be promoting breadth over depth in learning.

The use of the four branches is appropriate within middle years learning to structure the MYP Mathematics Skills Framework. There is no evidence to suggest that *Discrete mathematics* should be re-introduced as a fifth branch; indeed, this could be possibly counter-productive in that it may not allow for sufficient depth of learning within the other branches. In general, the majority of teachers are able to use the MYP Mathematics Skills Framework in its current form as a useful planning tool.

Of schools that deliver the full five years of MYP Mathematics, nearly 90 per cent agree that they aim to cover the majority of the suggested content. Approximately 80 per cent agree that the MYP Mathematics Skills Framework is supportive for planning within a year and across different years, and that the framework allows for effective progression to DP courses. The practitioner questionnaires suggest the greatest challenges to teachers and curriculum leaders may be how to use the MYP Mathematics Skills Framework to plan for making connections between mathematical ideas and embedding the subject content within the wider IB MYP philosophies.

Within the curriculum documents used in this study, there is a wide range of different structures and approaches to providing written guidance to support planning and learning. The approach taken by the MYP Mathematics Skills Framework is to provide lists of topics and skills as suggestions of possible suitable content. This is somewhat different to the majority of other systems, which are often more prescriptive in terms of what is expected of learners, with some even structuring learning year-by-year. In part, the approach taken by the IB, embedding the MYP Mathematics Skills Framework within the more comprehensive *MYP Mathematics Guide*, is a reflection of the need for MYP mathematics to be able to complement other systems and for schools to be able to use the MYP Mathematics Skills Framework flexibly according to their local circumstance.

NFER recommends the IB consider carefully the relative strengths of the structure of the current MYP Mathematics Skills Framework, in particular the scope it provides schools to plan their own curriculum alongside other state or national requirements, against the possibility that the MYP Mathematics Skills Framework does not provide the level of richness of subject guidance as some other systems (see Section 2.2). A possible solution may be to retain the current format and structure of the Mathematics Skills Framework but revise the content, and then also provide additional subject-specific guidance to provide greater richness and detail, as discussed in section 4.1.2. Whilst this may not be aligned to guidance documents for other MYP subject areas, there is evidence to suggest this may be needed by some practitioners to support effective curriculum planning and mathematics teaching in their schools.

NFER also notes that the IB has introduced an optional eAssessment qualification for its middle years mathematics programme and considers that the IB may benefit from ongoing research into the effects of the MYP eAssessment and associated assessment frameworks on the use of the MYP Mathematics Skills Framework within schools. In particular, NFER considers that the IB should monitor any changes in schools that are using the eAssessment and how they structure their mathematics curriculums to determine whether there is any impact on the breadth and depth of learning of mathematics in the middle years.

4.1.2 Quality of written guidance

The practitioner questionnaires and interviews indicate that the current Mathematics Skills Framework does not always provide sufficient written detail to support planning. Within the *Algebra* and *Geometry and trigonometry* branches, for each topic approximately 15 per cent to 20 per cent of teachers felt the current MYP Mathematics Skills Framework provided insufficient written guidance to support planning, with some topics gaining even higher percentages. In the *Number* and *Statistics and probability* branches, this figure was between 20 per cent and 25 per cent for each topic, with the vast majority of topics gaining over 10 per cent of respondents giving a more extreme negative rating.

For some teachers, there seems to be a challenge in linking the content within the MYP Mathematics Skills Framework to the wider MYP philosophies, and being able to use this to structure their own plans, as per the *MYP: From principles to practice*. The teacher questionnaire and expert panels suggested that part of this difficulty may stem from the challenge teachers face in dealing with the day-to-day job of teaching and having the time and opportunity to plan strategically. For some, additional support in sequencing and structuring mathematics learning may be beneficial. This could take the form of a 'Scope and Sequence' document as used in PYP, or use ideas from other systems such as the US Common Core State Standards or ACARA Frameworks to help provide a guide to support the effective structuring of mathematics learning. Alternatively, this support may be able to be developed through the IB Professional Development programme; this support mechanism is highly valued by MYP mathematics teachers. Prior to the development of any additional support, we would recommend the IB to undertake further research into whether there are any key demographic trends linked to teachers who do not consider the current MYP Mathematics Skills Framework to be supportive of their planning.

4.2 Revisions to the current topics and skills

4.2.1 Topics / skills to consider repositioning, removing or amending

We would recommend the IB revises the content specified in the MYP Mathematics Skills Framework. Table 3.1 indicates topics the IB should consider either repositioning in terms of the challenge level, removing from the MYP Mathematics Skills Framework entirely, or significantly revising the skills listed in the MYP Mathematics Skills Framework. From the curriculum comparison study, these topics / skills are generally not features of other middle years systems, and the Expert Panel feedback indicates that these are not topics that need to form a fundamental part of a middle years mathematics course. These topics also had the highest percentages (generally between 15 per cent and 20 per cent) of MYP teachers stating that they either do not include them in their current middle years mathematics courses, or consider them to not be appropriate to middle years mathematics.

Table 4.1 Topics to consider repositioning, removing or amending from current IB MYP Mathematics Skills Framework

Branch	Topic	Challenge level in current framework	Recommendation
Number	Number bases	E	Consider removing entirely.
Number	Logarithms	E	Consider removing entirely.
Number	Sets and Venn diagrams	S&E	Consider moving this topic to the Statistics and Probability branch, and update the skills to place emphasis on applying the ideas of set theory and Venn diagrams to solving conditional probability problems, as opposed to ‘formal’ set theory within the Number branch.
Algebra	Logarithms	E	Consider removing entirely.
Algebra	Arithmetic and geometric series	E	Sums of infinite series may be beyond the needs of middle years learners.

Branch	Topic	Challenge level in current framework	Recommendation
Algebra	Functions	S&E	Reduce the level of complexity surrounding trigonometric functions.
Algebra	Inequalities	E	Manipulating inequalities is appropriate within middle years mathematics. Linear programming is not a key feature of other systems. The IB should consider removing this to allow greater depth of study in other areas.
Algebra	Transformations of functions	S&E	Reduce the level of complexity of the examples within the skills.
Geometry and Trigonometry	Similarity and congruence	E	The skills listed in the current MYP Mathematics Skills Framework are appropriate to the *Extended* challenge level. We would also recommend adding this topic to the *Standard and extended* challenge level, with an emphasis on the links to proportionality, e.g. solving problems involving missing dimensions of similar figures where a scale factor can be identified.
Geometry and Trigonometry	Three-dimensional co-ordinate geometry	E	Consider removing entirely.
Geometry and Trigonometry	Vectors and vector spaces	E	Remove 'vector spaces' and 'dot product'.
Geometry and Trigonometry	Trigonometric identities	E	Some other systems for more-able middle years learners also include this topic, however it is not a part of the main systems analyzed. The IB should consider whether removing this topic would allow greater depth of learning elsewhere.
Statistics and probability	Population sampling	S&E	This topic may benefit from being expanded to consider the wider ideas of undertaking a statistical enquiry.

In terms of subject content, the *Extended* challenge level appears to be in greatest need of review. There is inevitable tension between providing a sufficient range of challenging mathematical ideas and topics to cater for the most able middle years learners but avoiding the promotion of breadth over depth. In its current form, at the *Extended* challenge level, it appears that breadth may be taking priority over depth. The topics listed above do not form part of the pre-requisite subject content requirements for the current DP Higher Level Mathematics Course. Although removing these topics may provide greater consistency between IB MYP mathematics and other middle years courses, and allow the focus for learning to be on depth of understanding of a narrower range of content, there could also be scope for the IB to consider moving these topics to an 'additional content' challenge level. As such, MYP practitioners would be able to see that these topics are not necessarily key features of middle years mathematics learning, or are topics that must be introduced within the middle years to secure progression to further study. Rather these topics could be drawn on as 'additional content' to inspire and challenge the most able learners, with schools selecting content appropriate to their learners and individual circumstances.

4.2.2 Possible additional topics

During all phases of this research, various topics and skills have been suggested as possibilities to include within a middle years mathematics programme. These may be dependent on local circumstance. Table 3.2 indicates the most commonly occurring suggestions, and our recommendations.

Table 4.2 Possible additional topics / skills to consider including in the MYP Mathematics Skills Framework

Topic / skill	Recommendation
Surds / radicals / irrational numbers	Other systems / curriculums explicitly state skills such as 'rationalizing the denominator'. Within the MYP Mathematics Skills Framework, this content is contained in the 'Forms of numbers' and 'Number systems' branch.
Rounding – levels of accuracy and relative errors	This topic was raised during the expert panel. Upper and lower bound analysis is a feature of other middle years systems and was considered appropriate content. Relative errors and error propagation may be potentially confusing to try to specify in a framework document.
Polar co-ordinates	This is not a feature of other middle years curriculums / systems. We would not recommend including it within the MYP Mathematics Skills Framework.
Statistical distributions	Understanding that data forms a distribution that can be analyzed is something that middle years learners should understand, and the idea of theoretical statistical models may provide a richer learning experience to some learners. Calculation of probabilities from binomial, normal or other

Topic / skill	Recommendation
	theoretical probability distributions lies beyond middle years requirements.
Randomness and variability	The existing MYP Mathematics Skills Framework specifies the calculation of probabilities. Understanding the idea of randomness and chance is an important aspect within *Statistics and probability*, and could be more explicit in the MYP Mathematics Skills Framework.
Permutations and combinations	Some other systems include the use of the product rule for enumerating possibilities. Within the probability sub-branch, it is appropriate for middle years learners to develop strategies for listing outcomes, e.g. using Cartesian products, however a more formal learning of permutations and combinations lies outside of the middle years.
Algebra - polynomials	The current MYP Mathematics Skills Framework specifies only consideration linear and quadratic forms. In terms of developing strong conceptual learning around algebra, an appreciation of polynomials may be important, however specific skills such as factorizing cubic forms or use of the factor / remainder theorems lie beyond middle years mathematics.
Constructions and loci	These topics could be suitable to include, provided the emphasis is on conceptual understanding. Compass-and-ruler constructions may allow for a rich deductive geometry system to be developed. Care would need to be taken that these topics did not simply form yet another procedural skill to develop.

In addition to the suggestions in Table 3.2, we would recommend the IB to consider the written guidance in the *Statistics and probability* branch. Current education thinking in this area places a greater emphasis on the ideas of planning and undertaking effective statistical enquiry, using and analysing data distributions and critiquing statistical reports in the media. Whilst many of these aspects of learning may be implicit in the overall *MYP Mathematics Guide* and MYP philosophies of learning, there may be scope to improve the written guidance in this branch. As outlined in Section 2.6, this branch has quite a wide variation across the other curriculums / systems and is also a source of some interesting innovative practice that warrants further research by the IB.

In addition to subject content revisions within the MYP Mathematics Skills Framework, we would recommend the IB to consider the *MYP Mathematics Guide* in full. Our research indicates that a key source of depth of learning in mathematics is not contained in the subject content itself, but rather in how learners are able to apply this content. On a global level, curriculum documents and expert opinion indicate that developing skills in problem solving and reasoning, in connecting different mathematical ideas together and in being able to reflect on strategies and solutions are important aspects of a deep understanding of mathematics. Middle years learners should develop skills in moving freely between the real world and the mathematical world, with learning being supported by technology where appropriate. Within IB MYP mathematics, these ideas lie outside of the scope of the MYP Mathematics Skills Framework itself but are key considerations in developing a middle years mathematics programme that meets the needs of future learners.

5 References

Alcantara, A. (2015) *IB Mathematics Comparability Study: Curriculum & Assessment Comparison - A Report for IB Global Recognition*. Cardiff: International Baccalaureate.

Australian Curriculum, Assessment and Reporting Authority (2015). *Learning Area*. Sydney: ACARA [online]. Available: http://www.australiancurriculum.edu.au/mathematics/structure [26 May, 2017].

Cohen, L., Manion, L. and Morrison, K. (2007) *Research Methods in Education.* (Sixth Edition). Oxford: Routledge.

Computer Based Maths Estonia (2013). 'Estonia named first computer-based math education country' (Press Release). *Computerbasedmaths.org*. [online]. Available: http://www.computerbasedmath.org/computer-based-math-education-estonia.php [2 June, 2017]

Consortium for Mathematics and Its Applications (COMAP) and Society for Industrial and Applied Mathematics (SIAM) (2016). GAIMME. *Guidelines for Assessment and Instruction in Mathematical Modelling Education*. USA: COMAP and SIAM [online]. Available: http://www.siam.org/reports/gaimme-full_color_for_online_viewing.pdf [2 June, 2017]

Erickson, L (2007). *Concept-Based Curriculum and Instruction for the Thinking Classroom.* (Concept-Based Curriculum and Instruction Series). London: Sage.

Greene, J. C., Kreider, H. and Mayer, E. (2005). 'Combining qualitative and quantitative methods in social inquiry.' In: Somekh, B. and Lewin, C. (Eds) *Research Methods in the Social Science.* London: Sage.

Hartas, D. (Ed) (2010). *Education Research and Inquiry*: *Qualitative and Quantitative Approaches.* London: Continuum.

International Baccalaureate (2016). The IB Middle Years Programme. Statistical Bulletin. June 2016 Examination Session [online]. Available: http://www.ibo.org/contentassets/4482e305ad0f43aea46f6b1eff78e3e5/myp-statistical-bulletin-june-2016-en.pdf [2 June, 2017].

Leonard, M. (2003). 'Interviews.' In: Miller, R., L. and Brewer, J. D. (Eds) *The A-Z of Social Research. A Dictionary of Key Social Science Research Concepts.* California: Sage.

6 Further reading

AQA (2017). *Level 2 Further Mathematics (8360)* [online]. Available: http://www.aqa.org.uk/subjects/mathematics/aqa-certificate/further-mathematics-8360 [20 June, 2017].

Australian Curriculum Assessment and Reporting Authority (ACARA) (2015) *Mathematics Learning area* [online] Available: http://www.australiancurriculum.edu.au/mathematics/rationale [20 June, 2017].

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2015). *Measurement Framework for Schooling in Australia* [online]. Available: https://acaraweb.blob.core.windows.net/resources/Measurement_Framework_for_Schooling_in_Australia_2015.pdf [20 June, 2017].

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2014). *Australian Curriculum* [online]. Available: http://www.australiancurriculum.edu.au/ [20 June, 2017].

Cambridge International Examinations (2017). *Cambridge IGCSE* [online]. Available: http://www.cie.org.uk/programmes-and-qualifications/cambridge-secondary-2/cambridge-igcse/ [20 June, 2017].

Computer Based Maths Estonia (2013). 'Estonia named first computer-based math education country' (Press Release). *Computerbasedmaths.org*. [online]. Available: http://www.computerbasedmath.org/computer-based-math-education-estonia.php [2 June, 2017]

Consortium for Mathematics and Its Applications (COMAP) and Society for Industrial and Applied Mathematics (SIAM) (2016). GAIMME. *Guidelines for Assessment and Instruction in Mathematical Modelling Education*. USA: COMAP and SIAM [online]. Available: http://www.siam.org/reports/gaimme-full_color_for_online_viewing.pdf [2 June, 2017]

Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA Center) (2017). *The US Common Core State Standards* [online]. Available: http://www.corestandards.org/about-the-standards/ [20 June, 2017].

Ministry of Education, New Zealand (2013). *The New Zealand Curriculum Online. Mathematics Standards* [online]. Available: http://nzcurriculum.tki.org.nz/National-Standards/Mathematics-standards [20 June, 2017].

Ministry of Education, Singapore (2012). Secondary Education: Syllabuses in Science Subjects (including Mathematics) [online]. Available https://www.moe.gov.sg/education/syllabuses/sciences/ [20 June, 2017].

Nuffield Foundation (n.d). *Key Ideas in Teaching Mathematics: Research-based guidance and classroom activities for teachers of mathematics* [online]. Available: http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics/ratio-and-proportional-reasoning [20 June, 2017].

Pearson Education Ltd. (2017). *Edexcel GCSEs* [online]. Available: http://qualifications.pearson.com/en/qualifications/edexcel-gcses.html [20 June, 2017].

Québec Government (2016). *Québec Education Program* [online]. Available: http://www1.education.gouv.qc.ca/sections/programmeFormation/index_en.asp [20 June, 2017].

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