Writing to enhance learning and inform teaching in mathematics

(An action research project undertaken with students and teachers at Year 6 of the Primary Years Programme)

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Executive Summary

ABSTRACT

This action research reports on the introduction of journaling in mathematics with a cohort of year 6 students (10-year and 11-year olds) engaged in their final year of the Primary Years Programme (PYP), within a large International Baccalaureate (IB) World School in Hong Kong. The project, carried out during the 2011-2012 academic year, sought to respond to the question: Does journal writing in mathematics support students to articulate their thinking and develop their conceptual understandings, while also serving to better inform teachers in order to quide planning? Data was collected at the beginning, midpoint and end of the study from both participating teachers and a sample of participating students. This was accomplished through informal group discussions in response to open questions posed by the researcher. Samples of student journals were also collected throughout the study for analysis. While the research findings were generally very positive, some points to note include: students for whom English was a second language found writing in mathematics challenging, whilst some other more mathematically able students just wanted to "get on with the math" and viewed journaling as holding them up. In summary, the rationale for introducing journaling as an aspect of a mathematics programme is strong. All participating teachers agreed that challenging students to better articulate their thinking and conceptual understandings in mathematics, as documented in the PYP *Mathematics scope and sequence*, not only develops metacognitive skills but also leads to opportunities for further inquiry, an important aspect of the Primary Years Programme philosophy.

INTRODUCTION

The concept of writing across the curriculum (WAC) is frequently credited to James Britton, a linguist from the University of London who explored the interdisciplinary relationship between language and thought. Britton's definition of writing across the curriculum explored *"the relationships between language and thought, between thinking and feeling, the links between unconscious and conscious ways of knowing, and the symbolizing nature of language."* (http://www.independent.co.uk/news/people/obituary-professor-james-britton-1428143.html). When we think of WAC in relation to mathematics it could be claimed that students have always written as part of their learning in mathematics. However, for the most part we would be referring to writing equations, answers and word problems as opposed to the type of writing described by Britton.

As a result of reading a range of relevant literature, and through personal experience, it is my belief that understanding tends to be more complete when one is required to explain, elaborate, or defend one's position to others. I would contend that this is especially so when operating in what is largely a non-verbal symbol system, as many perceive the abstract discipline of mathematics to be.

This action research project focused on writing *sensu lato*, i.e. in the broad sense. The goal was to introduce, implement and evaluate a variety of writing strategies and techniques as an integral part of the mathematics programme for a cohort of students in their final year of the PYP. The rationale upon which the research was based, was to support students so that they could articulate their learning and develop their conceptual understandings in mathematics, while also serving to better inform teachers in order to guide their planning.

The executive summary to follow reports the findings of the action research project from introduction, through implementation, to the end of the academic year, at which time the students who participated in the project exited the PYP. The cohort of 168 students represented the entire year level team of 6 homeroom classes, in an International Baccalaureate World School in Hong Kong.

THEORETICAL FRAMEWORK

The Primary Years Programme *Mathematics scope and sequence* document (International Baccalaureate, 2009) states that by communicating mathematically, both orally and in writing, students can explain and justify their thinking with and to others. This social constructivist approach, which is fundamental to the PYP, supports an interactive process whereby students create meaning by interacting and communicating with each other. It is important to note that communication, both oral and written, are equally acknowledged. The significance of communication in mathematics is further supported in *Making the PYP happen: A curriculum framework for international primary education*:

"In the PYP, mathematics is viewed primarily as a vehicle to support inquiry, providing a global language through which we make sense of the world around us. It is intended that students become competent users of the language of mathematics and can begin to use it as a way of thinking, as opposed to seeing it as a series of facts and equations to be memorized."

(Making the PYP Happen, 2009, Pg 81).

Other national curricula and mathematical organizations also support the importance of language in school mathematics programmes. The National Curriculum for England acknowledges that mathematics provides opportunities for pupils to:

"... develop the key skills of: communication, through learning to express ideas and methods precisely, unambiguously and concisely" (The National Curriculum for England, 1999, Pg 8).

The New Zealand curriculum includes the statement:

"Students learn mathematics through language and often must demonstrate their knowledge and understanding through language." (<u>http://nzcurriculum.tki.org.nz/NationalStandards/Mathematics-standards/Using-the-</u> mathematics-standards). The National Council of the Teachers of Mathematics (NCTM) includes in their Process Standards the process of Communication, which states that:

"the mathematical instructional programs from pre-kindergarten to grade 12 should enable all students to:

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teacher's, and others
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely." (http://www.nctm.org/standards/content.aspx?id=322)

METHODOLOGY

The term action research is often used by teachers and is frequently the research methodology of choice by those conducting research in a school setting. Kurt Lewin, a German-American social psychologist working at MIT (Massachusetts Institute of Technology) in the 1940's, is commonly referred to as the originator of the term action research and the methodology (Adelman, 1993). While there are many definitions of action research, the common features of most action research projects are that they are cyclic, participative, qualitative and reflective.

When reviewing the literature on action research in the context of education, a commonly cited model is that devised by Stephen Kemmis, which is cyclical and iterative, including 4 steps or stages:

> Plan Act Observe Reflect (Carr & Kemmls, 1986).



Figure 1: Action Research Model used

This was the model upon which the research described in this paper was informed and framed, in an attempt to respond to the following research question: *Does journal writing in mathematics support students to articulate their learning and develop their conceptual understandings, while also serving to better inform teachers in order to guide their planning?*

da Vinci style journaling (Appendix A) was introduced, practiced and refined throughout the action research process. Explained further, da Vinci style journaling is a form of visual journaling often used by artists, designers, inventors and others as a means to record thought processes, ideas, creative images, reflections and much more. It is suggested that Leonardo da Vinci always carried a journal with him; in fact, over seven thousand pages of da Vinci's journals still exist. Examples of other well known people who have kept visual journals include: artists Frida Kahlo and Edvard Munch, inventor Thomas Edison, musician Brain Eno, photographers Peter Hill Beard Peter and Dan Eldon, and scientist Stephen Hawking to name a few (Grauer & Naths, 1998).

This style of journaling was new to all involved in the research study, both teachers and students. The rationale for developing this style of journaling in mathematics was based upon the belief that writing in mathematics provides students with an opportunity to articulate their thinking and to reflect upon the solution path(s) they have pursued, in order to reach a mathematical conclusion or answer. This belief was further informed by the Visible Thinking approach developed through Project Zero at Harvard's Graduate School of Education (http://www.pz.gse.harvard.edu/visible_thinking.php). Student journal entries provided data, which was considered by the participating teachers and used to inform further practice.

Data for this study was gathered from group discussions, which were conducted with a random selection of students and all participating teachers. These group discussions were held at the beginning, middle and end of the project. The semi-structured discussions were video taped for reference and in order to produce a video documenting the project. Student journal samples were also shared and discussed by the group of participating teachers throughout the project. In this way, the data gathered was qualitative, participatory and reflective.

Narrative analysis and observer impressions were the main approaches used to analyse the data gathered, in order to identify patterns, themes and commonalities as well as contrasts.

FINDINGS & DISCUSSION

On three occasions during the course of the research project a small, random selection of eight students, from the total cohort of 168 students, and the six participating teachers were invited to share their thoughts, feelings and reflections on the project by responding to a number of general open-ended questions posed by the researcher. This was done in an informal group discussion context, and was recorded for later analysis and to produce a video for use with teachers as a professional development resource about writing in mathematics.

Initial Discussion

The first group discussions were done immediately prior to the start of the action research project.

Participating Teachers: (refer Appendix C1 for teacher questions)

The participating teachers were asked what motivated them to become involved in the project and about their previous experience with writing as an aspect of their mathematics programme. They were also asked to use a common thinking tool known as a PMI (positive, minus, interesting), devised by Edward De Bono and first published in 1982 in his book *De Bono's Thinking Course*, to share their thoughts about the action research ahead.

All 6 teachers were enthusiastic about being involved in the project and all saw the potential for including writing as an integral part of their mathematics teaching and learning programme. They were unanimous in their belief that the inclusion of writing in the math programme would support children's conceptual understandings and inform the teachers of individual student's strengths, weaknesses and needs. None of the participating teachers had included writing as an integral and regular part of their math programme before, beyond asking students to write brief reflections.

Initial thoughts and responses from the participating teachers about the project included:

"I've incorporated some writing in my mathematics before, the majority of it was reflections, more about the feelings that they (students) are feeling in mathematics and it was not really about their learning in mathematics."

"I haven't done math journaling before but I am just keen to see the impact it will have on students and to see also if it will affect my teaching in any way."

"Language has a key role within the PYP Mathematics scope and sequence, students are expected to use the language of mathematics in their work as well as explaining strategies throughout the different strands when they are solving problems."

"I've increasingly become aware that students really should have a voice, giving them a voice to think about their own learning, then to transfer that into giving them more ownership . . . the big positive could be getting students to think about their thinking."

"Writing in mathematics is not usually done, usually it is just numbers and equations, so writing about learning, I am interested to see how this will impact the understanding and learning of the students."

When asked what some of the potential negative and interesting points might be as a result of including writing within their math programmes, the teachers suggested that time may become an issue in an already busy teaching and learning schedule. Another common feeling was that writing might put off some of the students, especially those for whom both math and writing was personally challenging. In addition, one of the teachers feared that if she did not respond to students' journaling then her lack of response might impact upon the students' motivation to write.

"...if students are reflecting so much and journaling so much that I won't be able to get around all the time to make responses to them, I feel that might be an injustice to them... that they spend so much time on their journaling and I can't respond to them, they may kind of lose heart a little bit."

With regard to points of interest one teacher did mention the potential of including alternative methods of journaling, especially electronic or digital strategies and techniques, such as blogging and keeping a video diary.

Participating Students: (Refer to Appendix B1 for student questions)

A small group of eight randomly selected students were also involved in a group discussion immediately prior to the beginning of the research project. They were asked to share their feelings about the information they had been given by their teachers, i.e. that writing was to be included in the mathematics programme; whether they had ever done writing as part of mathematics before and if so in what form; and whether they felt that writing in mathematics could help them in their learning.

The eight students were all positively inclined toward the inclusion of writing as an aspect of their mathematics programme.

"I feel very excited and I'm looking forward to it."

"I think that doing writing in math will be very helpful because I can reflect on what I have done."

"I think it's going to be fantastic because it will allow us to reflect on our work."

None had any significant prior experience of writing in mathematics before, except for word problems and brief written reflections.

" I haven't done any math writing before but I'm really looking forward to it."

"I have never done any writing in math before."

" I've done some word problems before and some little reflections."

The students agreed that writing in mathematics might be helpful as a way of reflecting upon their work. When questioned further about what they meant by reflection in terms of mathematics, the students all articulated that it was describing what they had done and thinking about how they could improve next time. The students appeared to have a shallow understanding of what it meant to reflect; their description was more about recounting what had been done, identifying errors and highlighting ways to improve next time. Considering this common response by the students, I would suggest that it is difficult to improve if one hasn't moved into the cognitive realm of reflection, which, at this stage of the project, these students clearly had not.

When students were asked if they felt that writing in mathematics could help them in their learning they all felt that potentially it could.

" When we reflect we can look back on what we are doing wrong and do better."

" I think writing in mathematics could help us a lot because if we do something wrong then we can reflect on it and then the next day, we can come back and do that thing right because we have reflected on what we did wrong."

Midpoint Discussion

The midpoint group discussions were completed 10 weeks into the project. At this point writing and journaling had been introduced and included as an integral part of the mathematics programme, in all 6 participating classrooms. Refer to Appendix C2 for the questions posed in order to stimulate teacher discussion and Appendix B2 for the questions posed in order to stimulate student discussion.

Participating Teachers: (refer Appendix C2 for teacher questions)

To begin the discussion the teachers were asked to describe some of the techniques and strategies they had used to include writing as part of the mathematics programme during the first 10 weeks of the project. A range of strategies and techniques were described and identified strengths and weaknesses were commented upon.

Teachers commented that:

"I started off giving the students some techniques to start their reflections, using a diary entry or a reflection. One of the successful ones I used was a graphic organizer 'Two stars and a wish' where the students needed to write about two good things that happened that week in their math learning and then one thing that they wanted to work on . . . also using the online math blog with my class because they really enjoy using ICT and that's been a lot stronger than their writing in books."

"... the students responding to my teacher written prompts. Initially this resulted in a lot of writing about feelings but progressed to more descriptive writing about their learning. We moved on to more in depth writing, we looked at one concept area and then they would have to create a graphic organizer with different headings ... this is quite good because it meant that after learning for a couple of weeks they could go back to the graphic organizer and add to it with a coloured pencil and its showed their learning."

"... the children have written reflections in a math Blog on their own website . . . there's been real range of what the children have been doing, some children have really gone into depth about their math learning, others have been very simple in their responses and I think that reflects how much they value writing in math."

When asked if the inclusion of writing in the math programme had informed their teaching, and if so how, the 6 participating teachers were unanimous in their response that indeed it had.

"It's given me a better understanding of where the students are at in math. I've noticed that some of the more able students that really enjoy math are happy to write loads down and also type loads in their blogs, but on the other hand I've also noticed that some of my less able students who are quite reluctant to write . . . they will find graphic organizers quite easy to do and will use loads of diagrams."

"In a way it's allowed me to see things through the eyes of the students a lot more."

"... the inclusion of journaling in math's has helped me because I am not able to conference with all 28 students weekly, but using journaling has helped me understand how they think about math's and any of the problems or success they are having ... "

"... Its helped my own teaching because I've seen patterns where perhaps students haven't quite understood something and if I've seen in their journaling that some haven't understood the same concept then I know that it is something that I need to review ..."

" The writing has made me more aware of what the students are able to do . . . finding out that some of the content may be easy . . . also I am able to see gaps in learning from past years . . . "

"It's made me become more aware as a teacher as to what I need to do in my planning."

The midpoint discussion with teachers ended with an invitation to the teachers to offer suggestions for moving forward with the project.

"... they (students) are just going on about their feelings, I really want to go on to get them thinking about their thinking, the blue hat. "

"We've got to use technology and I also do have to remember that in my class I have some students who are not confident with English."

"... make a definite time throughout the week to set aside when you talk about the reflections and make it something that is part of a lesson not just an add on to a math lesson, it becomes part of the math learning."

All teachers agreed that there was a need to move from the affective to the cognitive domain and thereby get the students writing at a more conceptual and metacognitive level.

Participating Students: (Refer to Appendix B2 for student questions)

A sample group of eight participating students were asked to share the kinds of writing they had been introduced to in their math programmes over the preceding 10 weeks. As part of this discussion they were also asked, in turn, which two strategies or techniques they enjoyed most and why. They were then asked to follow on from this by sharing which two strategies or techniques they least enjoyed and why. To finish the discussion the students were asked, as were the participating teachers, if they had any suggestions for moving forward with the project.

The strategies shared by the students included: da Vinci style journaling; written reflections, voice thread which included both oral and written responses to teacher prompts or questions; blogs entries on both personal student and class websites; a variety of graphic organizers, personal notes to the teacher in their math journal; and exit notes completed at the end of mathematical inquiry.

The students all agreed that the strategies they enjoyed the most were those that were digital or electronic, for example VoiceThread (http://voicethread.com/) and digital blog entries and reflections (www.weebly.com). Several students mentioned that the blog reflections were good because they (students) could comment on what other students had written, answer other student's questions and give advice including feedback and feedforward. In this way the blog entries promoted collaboration and presented an opportunity for a social constructivist culture, whereby students were constructing shared meaning. The constructivist philosophy is the foundation upon which the PYP describes how children learn. *(Making the PYP Happen, 2009, Pg 6).*

Despite a keenness to use technology, pen and paper journal writing was also a favoured strategy with students, as the following comments highlight:

"... what works best for me is when I write in my math journal because I get to think best and when my math journal is in front of me I think deeply ... "

"... throughout the year I've really enjoyed writing notes to the teacher about how I feel about math because I can speak directly to the teacher ... "

"The da Vinci style journaling allows me to think creatively."

When asked about their least favourite strategies none of the students identified a specific strategy that they really did not enjoy doing. However all students interviewed were unanimous when asked for feed-forward. They all shared that they would like to use more electronic, digital journaling strategies in the future, i.e. VoiceThread and Weebly.

End of Project Discussion

Group discussions were also held at the end of the project. Refer to Appendix B3 for questions posed to students and C3 for questions posed to teachers in order to stimulate final discussions.

Participating Teachers:

When invited to reflect back over the project the teachers all agreed that the introduction of journal writing into their mathematics programme was worthwhile, both for themselves and their students. However teachers also identified that journaling did not work for all students. This was true for some students for whom English was a second language and for some of the more able mathematicians who felt that writing held them up. This was interesting in that it highlighted that for some students reflection was not a priority and was overlooked at the expense of getting finished.

All participating teachers agreed that they were motivated to continue to include some form of written journaling in their mathematics programme in the future. This unanimous support of the inclusion of writing in mathematics justified the action research project.

Participating Students:

When reflecting on the introduction of writing in mathematics, the students questioned all agreed that it had enhanced their ability to reflect with greater depth on their mathematical thinking. This was particularly pleasing since at the end of the first iteration of the action research model the participant teachers all agreed that there was a lack of metacognitive thinking coming through in student journal writing.

IMPLICATIONS AND RECOMMENDATIONS FOR PRACTICE

Overall the findings of this action research project were very positive in relation to the research question: *Does journal writing in mathematics support students to articulate their thinking and develop their conceptual understandings, while also serving to better inform teachers in order to guide planning?* All participating teachers agreed that student writing in mathematics did inform their teaching while also developing students' conceptual understandings. Student journal samples supported this, as did ongoing informal discussions with students.

Implications and recommendations for practice include the need to introduce and teach students about the concept of metacognition, specifically thinking about thinking mathematically. During the first iteration of the research model it became obvious that students had to be taught metacognitive strategies in order to make a shift from writing recounts describing what they had done in a math lesson, what they found easy, hard, enjoyable etc., to writing reflections that were more metacognitive and journal notes that were more analytical in relation to their mathematical thinking and conceptual understandings.

Time presented another challenge. Introducing a writing component into the math programme inevitably took time and this impacted both teaching and learning. For example, some of the more able students just wanted to *"get on with the math"* and viewed journaling as holding them up. While some students for whom English was a second language found writing in mathematics to be challenging and therefore more time consuming. The teachers who participated in the study raised the issue of the time journaling took in class and also the time required for them to review the student's journals. During the second iteration of the action research model, students were not required to write journal entries or reflections on a daily basis.

Another implication was the fact that this grade 6 cohort of students was part of a one-to-one laptop programme. Both participating teachers and students suggested that there were untapped opportunities for digital journaling that could be pursued in the future. Some of which were successfully implemented during the second iteration of the action research model, namely blogging and the use of VoiceThread.

CONCLUSION

In summary, the rationale for introducing journaling into the mathematics programme is strong. All participating teachers agreed that challenging students to better articulate their thinking and conceptual understandings in mathematics, as documented in the PYP *Mathematics scope and sequence*, not only develops student metacognitive skills in the context of mathematics, but also leads to opportunities for further inquiry, which is an important aspect of the Primary Years Programme philosophy. Capitalizing on digital and electronic journaling strategies opens further areas for research and inquiry.

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APPENDICES

- A. da Vinci style journaling student samples
- B. Student discussion questions
 - **B.1** Initial
 - B.2 Midpoint
 - B.3 End
- C. Participating teachers discussion questions
 - C.1 Initial
 - C.2 Midpoint
 - C.3 End











Appendix B: Student discussion questions

B.1 Initial (prior to the start of the project)

- 1. So your teacher has just told you that you are going to be doing writing in mathematics this year. What do you think about this?
- 2. Have you ever done any writing in math before? What kind of writing?
- 3. Do you think writing is part of mathematics?
- 4. How do you think writing about your learning in mathematics could help you?

B.2 Midpoint

- 1. What kind of writing have you done in mathematics so far this year? (group brainstorm)
- 2. Which two kinds of math writing have you enjoyed the most and why? (individual feedback)
- 3. Which kind of math writing have you enjoyed the least and why? (individual feedback)
- 4. What do you think we should focus on for the next 2 months in math journal writing? Can you suggest any changes?

B.3 End of project

1. Reflecting on the writing in mathematics focus that we have had this year what are your overall thoughts (strengths, weaknesses)

Appendix C: Participating teachers discussion questions

- C.1 Initial (prior to the start of the project)
- 1. What originally motivated you to become involved in the *Writing to enhance learning and inform teaching in mathematics* action research project?
- 2. Have you incorporated writing as an aspect of your mathematics programme before? What kind of writing? How successful was it?
- Thinking about writing in mathematics using the Plus/Minus/Interesting (PMI) mind tool:
 P = What advantages or positive outcomes can you foresee form your mathematics teaching and learning programme? For your students learning?
 M = What possible negative's do you foresee?
 I = Can you consider any interesting points that do not obviously fall into the positive or negative category?
- C.2 Midpoint
- 1. Can you describe some of the techniques and strategies you have used to include writing as part of your mathematics programme thus far? Including the strengths and weaknesses.
- 2. Has the inclusion of writing in your math programme informed your teaching? If so in what way?
- 3. Moving forward what suggestions do you have?

C.3 End

- 1. Reflecting on the project did journaling work for all students and if not what were some of the reasons?
- 2. Will you continue to include some form of writing and/or journaling in your mathematics programme?